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Stabilizing traffic with autonomous vehicles

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***Abstract*— Autonomous vehicles promise safer roads, energy savings, and more efficient use of existing infrastructure, among many other benefits. Although the effect of autonomous vehicles has been studied in the limits (near-zero or full penetration), the transition range requires new formulations, mathematical modeling, and control analysis. In this article, we study the ability of small numbers of autonomous vehicles to stabilize a single-lane system of human-driven vehicles. We formalize the problem in terms of linear string stability, derive optimality conditions from frequency-domain analysis, and pose the resulting nonlinear optimization problem. In particular, we introduce two conditions which simultaneously stabilize traffic while imposing a safety constraint on the autonomous vehicle and limiting degradation of performance. With this optimal linear controller in a system with typical human driver behavior, we can numerically determine that only a 6% uniform penetration of autonomously controlled vehicles (i.e. one per string of up to 16 human-driven vehicles) is necessary to stabilize traffic across all traffic conditions.**

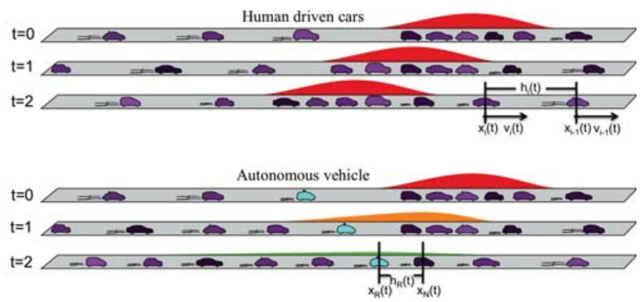

Fig. 1. In freeway traffic, human drivers respond to their preceding vehicles; human reaction times along with environmental influences can cause a backwards propagating shockwave of slower vehicles (traffic jams indicated by the red). An autonomous vehicle can issue precise control by considering the overall traffic conditions and models of the human drivers, thereby dampening the shockwave. Vehicles following the autonomous vehicle then experience an attenuated perturbation. (Image courtesy Florian Brown-Altvater)

I. Introduction

Transportation accounts for 28% of energy consumption in the US. 75% of that occurs on highways—more if we include major arterials. Workers spent on aggregate over three million driver-years commuting to their jobs [1], with significant impact on nation-wide congestion. Based on 2012 estimates, U.S. commuters experienced an average of 52 hours of delay per year, causing $121 billion of delay and fuel costs annually [2], and estimates project that 4.2% of fuel will be wasted in congestion in 2050 (up from 1.8% in 1998) with the adoption of autonomous vehicles [3].

Traffic jams are undesirable—for vehicular throughput, driver safety, energy consumption, etc.—and this article investigates the potential for using autonomous vehicles to mitigate these jams. Human drivers can sometimes spontaneously cause stop-and-go traffic conditions—remarkably, even without provocation [4]. This comes from a property known as *string instability*, where some perturbations get amplified as they pass down a singly connected chain of (plant stable) vehicles [5]. This causes uniform flow to be an unstable equilbrium, with natural disturbances quickly amplifying into high amplitude stop-and-go shock waves [6].

In this article, we use autonomous vehicles to induce string stability in a system of (human-driven) vehicles that is otherwise string unstable, attenuating perturbations and thereby preventing the formation of traffic jams. Our problem setting and motivation is depicted in Figure 1.

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In addition to considering (asymptotic) stability, we must additionally maintain a safety condition, ensuring that the autonomous vehicles never get too close to their preceding vehicle. We may also like to define an efficiency criterion, where the autonomous vehicles also do not lag too far behind their preceding vehicles. Together, these conditions form an optimization problem; its solution gives a linear controller corresponding to the optimal autonomous vehicle which permits a safe and performant stabilization of uniform traffic flow at the lowest penetration rate of autonomous vehicles. Through numerical analysis, we can determine this optimal autonomous controller given a specific human car following model.

In particular, the contributions of this paper include:

- A microscale (vehicle-level) transfer function formulation of traffic jam formation in single-lane traffic,
- Analytic conditions on this formulation characterizing (string) stability, safety, and relative performance of autonomous vehicle controllers in a string of human drivers,
- An optimization problem defined by those conditions to minimize the necessary penetration rate of autonomous vehicles, and
- Numerical analysis of the optimization problem and its resulting linear controller on standard traffic models.

We find that with representative human driver behavior, a single vehicle using our autonomous controller can stabilize a string of up to 16 human driven cars across all traffic conditions (i.e. at a uniform 6% penetration). With even fewer autonomous vehicles, we can still improve overall driving conditions, stabilizing uniform flow at a subset of previously unstable traffic conditions. This autonomous controller is robust to small variations in human driver model parameters. The necessary stabilizing penetration rate is sensitive to one of the autonomous controller parameters, but robust to small changes in the others.

II. Related Work

A. Dynamics of road traffic

Numerous car following models (CFMs) have been developed to simulate longitudinal road traffic, specifying the dynamic trajectory of a vehicle in relation to the vehicle preceding it [7]. The Intelligent Driver Model (IDM) [8] is a widely used nonlinear model for evaluating human behavior as well as for implementing adaptive cruise control (ACC) [9]. Other models are also in use, guaranteeing collision-free dynamics [10] or providing analytically tractable equations [6]. The concept of string stability has been introduced to model and analyze the naturally occurring stop-and-go waves in traffic for CFMs [11]. In this article, we focus on a single-lane highway setting, using IDM in our work to represent longitudinal traffic dynamics. Additionally, we extend the string stability analysis to heterogeneous models and apply the concept to traffic control.

B. Vehicle control

In response to traffic patterns (in particular anticipative traffic), several controllers have been proposed to optimize driving ease, comfort, and efficiency for the individual controlled vehicles [12]. Examples include adaptive cruise control (ACC) and more recently, cooperative adaptive cruise control (CACC). ACC automatically adapts the cruise-control velocity of a vehicle if there is preceding traffic to a safe following distance [13]–[15]. CACC extends ACC with a wireless intervehicle communication link, which permits the use of more observations (from other vehicles) for more precise control [16]–[19].

C. System-level impacts of autonomy

It has been demonstrated that human drivers [4] and imperfect autonomous controllers [20] may cause upstream shockwaves of heavy braking in a string of vehicles, forming traffic jams throughout the system despite the absense of accidents or bottlenecks. This string-instability can also be observed in consecutive vehicles using ACC, which amplify the speed variations of preceding vehicles [9]. On the other hand, consecutive vehicles using CACC have been shown to prevent the local formation such shockwaves in those vehicles in single-lane experiments [9], [21]. In this article, we are concerned with autonomous controller design which optimizes global system-wide metrics without requiring vehicle-to-vehicle (V2V) connectivity.

A related study characterizes a variety of systems across autonomy type, control parameters, and penetration rates within the framework of string stability [22], while our work poses an explicit optimization problem to identify control parameters to realize the best achievable penetration rate.

D. Autonomous fleets

The vast majority of work on system-level effects—e.g. energy consumption, total travel time, and string stability—of autonomous vehicles consider either close to full penetration as to attain full control [3], [23]–[25], or small enough of a penetration as to not affect traffic dynamics [26], [27].

In this work we bridge the gap between minimal and full penetration of autonomous vehicles by studying a mixed autonomy setting, a setting that is projected to be reality for at least the next 35 years [3]. A few studies have begun considering the mixed setting, modeling and simulating a mixture of ACC and manual vehicles [28], [29]. Our work is most closely related [30], [31] which demonstrates autonomous controllers that eliminate traffic jams at low penetration rates from specific unstable ring road configurations.

In contrast to these, our paper provides general analysis, derivation of optimality conditions, and an explicit optimization problem on system-wide string stability while maintaining vehicle-level considerations including safety and efficiency.

III. Preliminaries

A. Microsimulation modeling

Standard car following models (CFMs) are of the form:

$$a_i = \dot{v}_i = f(h_i, \dot{h}_i, v_i), \tag{1}$$

where the acceleration a_i of car i is some typically nonlinear function of $h_i, \dot{h}_i, v_i$, which are the headway, relative velocity, and velocity for vehicle i, respectively. Though a general model may include time delays from the input signals $h_i, \dot{h}_i, v_i$ to the resulting ouput acceleration a_i, we will consider a non-delayed system, where all signals are measured at the same time instant t. Example CFMs include the Intelligent Driver Model (IDM) [8] and the Optimal Velocity Model (OVM) [32], [33].

In a uniform flow equilibrium, each car moves at a constant velocity v^* with constant headway h^*. We use the term *traffic condition* to refer to this equilibrium velocity v^*. It is intuitive to consider the equilibrium velocity relative to a target velocity v_0 (free flow speed), comparable to a speed limit for highway traffic [8]. In practice, the equilibrium can be determined or estimated by the local traffic density. In settings with heterogeneous vehicle types, the equilibrium can be numerically solved by constraining the total headways to be the total road length and the velocities to be uniform.

At this uniform flow equilibrium, we have

$$a_i = 0 = f(h^*, 0, v^*), \tag{2}$$

defining the relationship between the two equilibrium quantities h^*, v^*. To characterize this system, we consider the linearization of the dynamics about the equilibrium,

$$a_i \approx \partial_h f|_{eq} \cdot (h_i - h^*) + \partial_{\dot{h}} f|_{eq} \cdot \dot{h}_i + \partial_v f|_{eq} \cdot (v_i - v^*) \tag{3}$$

with coefficients $k_p = \partial_h f|_{eq}$, $k_d = \partial_{\dot{h}} f|_{eq}$, $k_v = -\partial_v f|_{eq}$.

Let $x_i(t)$ be the absolute position of vehicle i at time t, and let $\tilde{x}_i(t)$ be the position of vehicle i relative to its equilibrium position at time t. The resulting linear dynamical system can be re-written as

$$\ddot{\tilde{x}}_i = k_p(\tilde{x}_{i-1} - \tilde{x}_i) + k_d(\dot{\tilde{x}}_{i-1} - \dot{\tilde{x}}_i) - k_v(\dot{\tilde{x}}_i) \tag{4}$$

using suitable transformations and initial conditions:

$$x_i(0) := -ih^* \quad \forall i \tag{5}$$

$$h_i(t) := x_{i-1}(t) - x_i(t) \quad \forall i \in \{1, \dots\} \tag{6}$$

$$\tilde{x}_i(t) := x_i(t) - x_i(0) - tv^* \quad \forall i. \tag{7}$$

For convenience of notation, we have not explicitly included the length of the vehicles L_{veh} in these equations, but it should be straightforward to see how this definition of headway maps to the bumper-to-bumper headway.

We now denote $T(s)$ the transfer function for the linear dynamics of Equation (4), with $\tilde{X}(s)$ the laplace transform of $\tilde{x}(t)$. For vehicle i:

$$T_i(s) := \frac{\tilde{X}_i(s)}{\tilde{X}_{i-1}(s)} = \frac{k_{di}s + k_{p_i}}{s^2 + (k_{di} + k_{vi})s + k_{p_i}}. \tag{8}$$

For the remainder of this article, we use subscript $_H$ (respectively $_R$) for the index i to differentiate between the linearized dynamics of a nonlinear human car following model (respectively autonomous vehicle controller).

B. System characteristics

We study the linear stability of the uniform flow [6] (also called string stability or platoon stability [34]). It should be noted that car-following dynamics cannot be fully captured through linearization and non-linear stability analysis should be performed, especially when not close to the system equilibria. Unfortunately, except for limited models, non-linear stability analysis is not analytically tractable. However, recent work has demonstrated the existence of vehicle controllers which can bring the overall mixed-autonomy traffic system system close to uniform flow equilibria, the regime where the following analysis and optimization is valid [31].

Although there is ambiguity present in the literature, the principle behind string stability is that an oscillation experienced by a vehicle (e.g. at $i = 0$) should be attenuated upstream a string of vehicles (e.g. $i > 0$) [21].

Definition 1 (Vehicular string stability [21], [35]): A vehicle with transfer function $T(\cdot)$ is string stable if and only if $|T(j\omega)| \leq 1, \forall \omega$. Equivalently, $\|T(j\omega)\|_\infty \leq 1$.

From [6], [34], we have another condition for string stability for the linearized dynamics of the non-delayed system under oscillatory perturbations at frequency ω.

$$k_p \leq \frac{1}{2}\omega^2 + k_d k_v + \frac{1}{2}k_v^2, \quad \forall \omega \in \mathbb{R}^+. \tag{9}$$

Definition 2 (System-level string stability): Let $(T_i)_{i\in[N]}$ denote the transfer functions of the N vehicles in a single lane traffic system. This system is string stable if and only if

$$\left\| \prod_i T_i(\cdot) \right\|_\infty \leq 1. \tag{10}$$

The main restriction from the definition of [36] is that the length of the string is given.

Note that the stability of a system as defined does not prohibit collisions between vehicles or inefficient driving behavior. This motivates additional metrics on vehicles in the traffic system.

Definition 3 (Headway bounds): We define the *safety bound* $\Delta_- > 0$ as the minimum headway that a vehicle is allowed to experience. That is, regardless of any given external disturbance, we require the headway to maintain

$$h(t) > \Delta_- \quad \forall t > 0 \tag{11}$$

Similarly we specify the *performance bound* as the maximum allowable headway $\Delta_+ > 0$ that a vehicle can permit:

$$h(t) < \Delta_+ \quad \forall t > 0. \tag{12}$$

By bounding the headway below, we ensure safety by maintaining separation between vehicles. By bounding its maximum value, we ensure that the vehicle does not lag too far behind the rest of the traffic. Together, these relations impose an allowable region for a vehicle's headway relative to its equilibrium, which we can collect into a single *headway bound* $\Delta > 0$:

$$\Delta_- = h^* - \Delta < h(t) < h^* + \Delta = \Delta_+ \quad \forall t > 0. \tag{13}$$

IV. Optimization Problem Formulation

A. Overview

With these definitions in place, we can now find and characterize an autonomous vehicle controller to stabilize a homogenous system of human vehicle models, without degrading performance or violating safety. In particular, given an otherwise string unstable human traffic system, we aim to minimize the density of autonomous vehicles necessary to render that system string stable. Equivalently, for a *single* autonomous vehicle, we aim to find and characterize the maximum number of human vehicles n^* it can stabilize, safely and efficiently. We will consider this second setting for the remainder of the article, posing the resulting nonlinear optimization problem by formalizing the constraints and optimality conditions.

We make several assumptions and reductions in our work:

- Human vehicle models are homogeneous—all human-driven vehicles can be approximated by the same vehicle dynamics,
- Human vehicle models are string unstable in some traffic conditions—there exists an equilibrium point such that the linearized coefficients of the model (Equation (3)) fail to satisfy inequality (9) for some perturbation frequency ω,
- All vehicle models are plant stable—bounded disturbances in headway experienced by the vehicle do not result in unbounded deviation from its equilbrium position, and
- The traffic system is confined to a single-lane ring road setting and to models without explicit time delays [37].

B. Optimality conditions

It will be convenient to note the following fact, which can be algebraically derived by evaluating the derivatives of $|T(jw)|$.

Lemma 1 (Critical frequency range): Given linear human vehicle model (k_p, k_d, k_v), let

$$\omega_0 := \sqrt{k_p - k_d k_v - \frac{1}{2}k_v^2} \tag{14}$$

If the system is string unstable, then $|T(j\omega)| \geq 1$ for $\omega \in [0, \sqrt{2}\omega_0]$, with equality at the endpoints, and is stable otherwise. Furthermore, $\|T(j\omega)\|_\infty$ occurs at $\omega = \omega_0$.

With this, we can formulate our optimality conditions for autonomous vehicle controllers.

Theorem 2 (Stability condition): The maximum number of vehicles a single autonomous vehicle can string stabilize is given by

$$n^*_{stable} = \left\lfloor \min_\omega -\frac{\log|T_R(j\omega)|}{\log|T_H(j\omega)|} \right\rfloor, \tag{15}$$

over ω in the range where the human drivers are string unstable, i.e., $T_H(j\omega) > 1 \implies 0 < \omega < \sqrt{2}\omega_0$. Outside this range, the system is trivially string stable for sufficient density of human driven vehicles.

Proof: Consider a string of n human driven cars followed by a single autonomous vehicle (though the order of the vehicles does not matter due to the linearity of the scalar systems). Label the position of the autonomous vehicle (relative to its equilibrium position) $\tilde{x}_R(t) \rightsquigarrow \tilde{X}_R(s)$. Furthermore, consider that the behavior of the leading (human driven) car is governed by its reaction to some input (disturbance) $d(t) \rightsquigarrow D(s)$. Then, from Equation (8), we get the relationship:

$$\tilde{X}_R(s) = T_R(s)T_H(s)^n D(s). \tag{16}$$

By Definition (2), this system is string stable if it attenuates all unstable disturbances; that is, for $s = j\omega$, $0 < \omega < \sqrt{2}\omega_0$:

$$\begin{aligned} |T_R(s)T_H(s)^n| &\leq 1 & (17)\\ \log|T_R(s)| + \log|T_H(s)^n| &\leq 0 & (18)\\ n &\leq -\frac{\log|T_R(s)|}{\log|T_H(s)|} & (19) \end{aligned}$$

The maximum number of human driven cars that can be stabilized under any input disturbance by a given autonomous car controller $T_R(\cdot)$ is then given by the minimum of Equation (19) across all unstable disturbances, giving our result. ■

Theorem 3 (Safety/performance condition): For a given scaled step perturbation $d(t) = \beta u(t)$ and headway bound Δ, the maximum number of vehicles a single autonomous vehicle can safely and efficiently follow is given by

$$n^*_{safe} = \left\lfloor \min_\omega \frac{\log\eta - \log|1 - T_R(j\omega)|}{\log|T_H(j\omega)|} \right\rfloor, \tag{20}$$

with $\eta := \frac{\Delta}{\beta}$ and $0 < \omega < \sqrt{2}\omega_0$, as above.

Note that we are considering a step disturbance here for ease of notation and analysis, but the theorem holds for oscillatory and impulse disturbances as well, and is thus likely to cover the extremes of realistically encountered perturbations.

Proof: As in the previous proof, we can further consider the position of the final (human driven) car in the string $\tilde{x}_N(t) \rightsquigarrow \tilde{X}_N(s)$:

$$\tilde{X}_N(s) = T_H(s)^n D(s), \tag{21}$$

giving rise to the headway experienced by the autonomous vehicle $h_R(t) - h^* = \tilde{x}_N(t) - \tilde{x}_R(t)$. That is,

$$\tilde{X}_N(s) - \tilde{X}_R(s) = T_H(s)^n(1 - T_R(s))D(s). \tag{22}$$

As these are the final two cars in the traffic string, they experience the greatest disturbance, having been amplified along the chain of string-unstable human drivers. If the autonomous vehicle were earlier in the string, it would experience a smaller headway deviation. Thus, this guarantees that the autonomous controller does not worsen safety or efficiency from an uncontrolled, fully human model.

Then, from Definition (3) and algebra, and noting the equivalence of ∞ signal and system norms (max values) under these disturbances,

$$\begin{aligned} |T_H(s)^n(1 - T_R(s))\beta\frac{1}{s}| &< \Delta\frac{1}{s} & (23)\\ \log|T_H(s)^n| + \log|1 - T_r(s)| &< \log\left(\frac{\Delta}{\beta}\right), & (24) \end{aligned}$$

and the conclusion immediately follows. ■

C. Optimization problem

Now we present our optimization problem, which selects autonomous vehicle parameters which are admissible under both optimality criterion. Given linear human vehicle model T_H and safety parameter η, we would like to optimize for

$$\begin{aligned} n^* &= \max_{T_R} \min(n^*_{stable}, n^*_{safe}) & (25)\\ \text{s.t.}\quad T_R(s) &= \frac{k_{d_r}s + k_{p_r}}{s^2 + (k_{d_r} + k_{v_r})s + k_{p_r}} & (26) \end{aligned}$$

Definition 4 (Penetration rate): Given the traffic condition v^* defining the human CFM T_H, the penetration rate

$$\rho = \frac{1}{n^* + 1} \tag{27}$$

describes the fraction of uniformly distributed autonomous vehicles (ensuring one autonomous vehicle per n^* human vehicles) necessary to string-stabilize the heterogenous traffic system at this equilibrium—preventing the formation of stop-and-go traffic through the attenutation of disturbances—with a bounded decay in performance given bounded perturbations, where n^* is the optimal objective value of Problem (25).

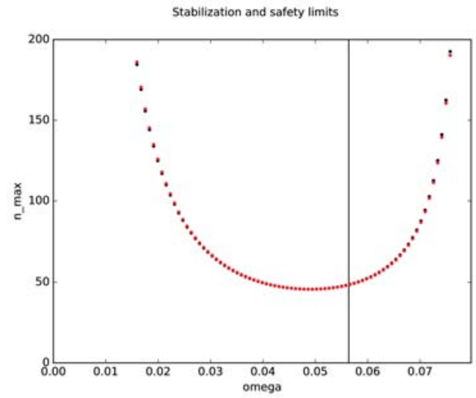

Fig. 2. For the given human vehicle model and the optimal autonomous vehicle controller, we vary ω within the unstable region of frequencies (for the human vehicle) and present the number of vehicles that can be stabilized (black) or safely handled (red) by the autonomous vehicle. The two curves are nearly identical, demonstrating that the optimization procedure successfully chose parameters that optimized for both conditions. The vertical line denotes the most unstable frequency for the human vehicle model ω_0, so we observe that adding the autonomous vehicle shifts the critical frequency to the left.

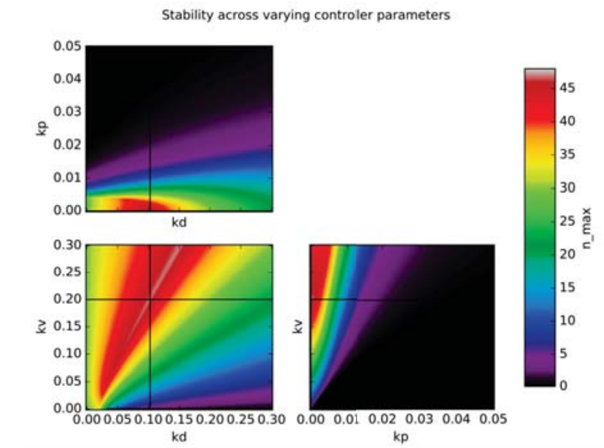

Fig. 3. For the fixed default human vehicle model, two of the three autonomous vehicle parameters $k_{p_r}, k_{d_r}, k_{v_r}$ are varied while the last parameter is held constant to the optimal autonomous vehicle controller, producing a heatmap of the number of human vehicles a single autonomous vehicle can safely and efficiently stabilize.

V. Numerical Results

We start by considering a human IDM driver with parameters $v_0 = 33m/s, T = 1.5s, s_0 = 2m, a = 0.3m/s^2, b = 3m/s^2$ [38], in traffic conditions defined by $v^* = v_0/2$. This defines the human car following model of Equation (1) that we are using, and gives a linearization with $k_p, k_d, k_v = 0.01, 0.18, 0.04$.

We use physical constraints to motivate our numerical setting, restricting our search space over the autonomous vehicle controller to physically realizable controllers. It is likely that the largest perturbation in headway that a vehicle will experience results from lane changing. Typical lane changes occur at headways of 100 m (in) or 70 m (out) [39], thereby introducing a perturbation $\Delta h_{max} \approx 50 - 70m$, or $\eta < 2$. When approaching a stalled car or traffic accident from free flow at $v_{max} = 40m/s$, we see a maximum rate of change of headway at $\dot{h}_{max} \approx v_{max} = 40m/s$. Similarly, when considering accelerating from a dead stop onto an empty highway with free flow speed $v_{max} = 40m/s$, we see a maximum velocity difference $\Delta v_{max} \approx v_{max} = 40m/s$.

Human drivers comfortably accelerate at up to $a_{max} \approx 0.5$ gee, with slightly higher tolerance for braking than accelerating [40]. This then gives us maxima:

$$k_p \approx \quad a_{max}/\Delta h \quad < 0.1, \tag{28}$$
$$k_d \approx \quad a_{max}/\dot{h}_{max} \quad < 0.2, \tag{29}$$
$$k_v \approx \quad a_{max}/\Delta v_{max} \quad < 0.2. \tag{30}$$

We therefore cap our $k_v < 0.2$, giving us an optimal autonomous vehicle controller of $k_p, k_d, k_v = 0.000, 0.103, 0.200$.

At this T_H, T_R, you can see how n^*_{stable}, n^*_{safe} changes across the range of unstable perturbation frequencies ω in Figure 2. The optimal controller lies along the asymptote $k_v = \eta k_d$ of locally optimal autonomous vehicle controllers, as can be seen through the effects on n^* as we vary the parameters of T_R in Figure 3. This controller is robust to variations in human driver model as well, as shown in Figure 4.

Finally, we can consider the system level traffic perspective through the fundamental diagram in Figure 5. As we vary the density of traffic in a system defined by the given IDM human driver model, the uniform flow equilibrium of Equation (2) defines the resulting flow rate at that equilibrium. In order to achieve this flow rate though, the system must be string-stable at that equilibrium point; the formation of stop-and-go traffic waves would otherwise lower the resulting flow rate. Thus we can use our problem formulation to evaluate our optimum autonomous controller to string-stabilize the traffic system and achieve efficient behavior. As we vary the traffic conditions, a minimum uniform penetration rate of $\rho = 6\%$ autonomous vehicles is necessary to stabilize the uniform flow equilibrium in all situations. At lower penetration rates, denser traffic conditions will still result in stop-and-go traffic, but the range of string stability is nonetheless increased from purely human traffic.

VI. Conclusions and Discussion

In this article, we present the first analysis of the penetration rate of autonomous vehicles needed to stabilize traffic with bounded degradation of performance and safety properties. We derive system-level string stability, safety, and performance conditions using linearized frequency-domain analysis, and we pose a nonlinear optimization problem to solve the multi-objective problem. Numerically, we show that for typical highway traffic conditions, only a 6% uniform penetration rate of autonomous vehicles can safely and efficiently stabilize traffic under all uniform flow conditions.

The optimal controller so discovered lies at $k_p = 0$, defining an autonomous vehicle controller that is nonresponsive to the vehicle's headway. This is an unreasonable condition

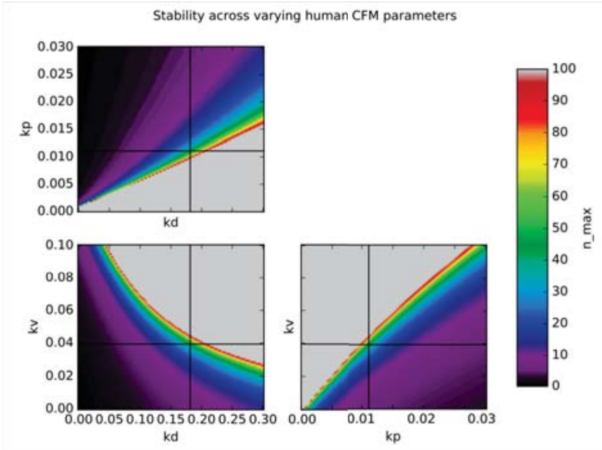

Fig. 4. For the optimal autonomous vehicle controller, two of the three human vehicle parameters k_p, k_d, k_v are varied while the last parameter is held constant to the default human model, producing a heatmap of the number of human vehicles a single autonomous vehicle can safely and efficiently stabilize. The vertical and horizontal lines denote the default human model. We cap values, so the gray region denotes either stable parameters or models that permit more stabilized vehicles than the scale. We observe that there is a graceful decay of n^* as the human parameters vary, and in fact the varying parameters may be beneficial as well.

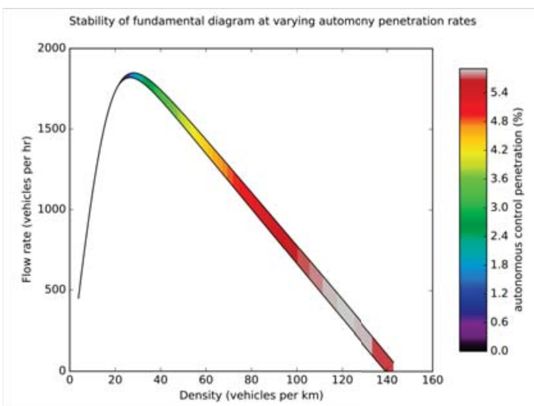

Fig. 5. The number of human cars a given autonomous vehicle can safely stabilize varies with traffic conditions. Thus, as the penetration rate of autonomous vehicle in a human traffic flow increases from 0, a greater range of equilibrium points of uniform flow become stable. In the above fundamental diagram, the width and color of the line both represent the minimum uniform penetration rate of autonomous vehicles necessary to stabilize that traffic regime. Though we need at least 6% of the cars in this example to be autonomous to stabilize all flows, fewer than 3% penetration still results in substantial improvements in stability up to and beyond the cusp of the curve.

which may result in unsafe behaviors in the presence of extreme disturbances outside the scope of this paper but potentially visible in actual traffic systems. As such, we can mandate $k_p > 0$ in our optimization constraints; this pegs the results of the optimal controller to the minimum allowable k_p as demonstrated by Figure 3. Furthermore, to handle this and other extreme road conditions, the autonomous controller will have to exhibit nonlinear behaviors. We can apply the derived linearized parameters to a human driving model, e.g. OVM, to give a better grounded robot controller, or we can optimize over such driver model parameters directly. This and similar extensions of the optimization problem presented in this paper are under current research.

Finally, with the potential availability of greater sensing capabilities, including the state measurement of multiple nearby vehicles, much more sophisticated controllers may be possible to further reduce the penetration rates necessary to stabilize human traffic flows. Nonetheless, this preliminary work shows the feasibility of a simple linear controller, extending basic ACC principles, to have significant impact in traffic systems.

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