What is the surface tension of this kitty?

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***Abstract*—Cats are often observed in puddle form as they lie on the floor. Key parameters relating to this feline fluid can be extracted from the resulting geometry. Fitting an empirical curve around the volume of a kitty-puddle allows us to calculate its surface tension. This combined with the contact angle of the puddle with respect to the floor gives us the energy of adhesion of the cat-floor interface. Analysis of a representative sample kitten on a rather ailurophobic hardwood floor with contact angle of 130^o indicates a surface tension of $30N/m$. This calculated value is over an order of magnitude greater than any conventional fluid.**

***Index Terms*—Feline fluids, Kitty-puddle, Surface tension, Adhesion energy, Empirical model**

Fig. 1. What is the surface tension of this kitty?

I. Introduction

Puddles are formed when a liquid comes in contact with solid surface in a gaseous medium. Counteracting forces define the geometry of this sessile drop, both at the liquid-gas interface and the liquid-solid-gas triple point. The cohesive force between liquid molecules draws the drop together while body forces such as gravity pull the molecules towards the solid-liquid interface. Adhesion between the liquid and the solid spreads the drop out.

Known parameters can be applied to generate the resultant puddle shape. Conversely, an empirical match of a puddle's geometry to one of a family of such shapes can be used to infer the fluid parameters. This study seeks a numerical approximation to feline surface tension σ and adhesion energy per unit area to a wooden floor ΔW in air. The representative kitty-puddle to be analyzed is shown in figure 1 [1].

II. Background

A. Bond Number

The Bond number Bo is a dimensionless ratio of body (gravitational) effects to cohesive (surface tension) forces of a liquid in a gaseous medium. The gravitational forces are parameterized by the densities of the fluids ρ along with a characteristic physical dimension r_0, the radius of curvature at the top of the liquid drop. The cohesive force is represented by the surface tension of the fluid-gas interface σ [2].

$$Bo = \frac{g r_0^2 \left(\rho_{liq} - \rho_{gas}\right)}{\sigma}. \tag{1}$$

A Bond number of zero, indicating no body forces at all, results in a perfectly spherical liquid-gas interface. Higher Bond numbers indicate greater influence of gravity (or lower surface tension), causing flattening of the bubble. The specific shape of the drop can be numerically determined and tabulated for a range of Bond numbers [3]. A comparison of puddles with $Bo = 1, 5$ is shown in figure 2.

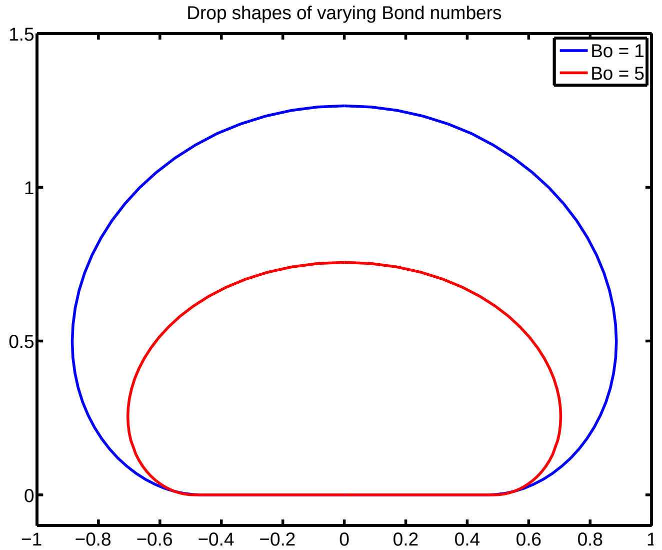

Fig. 2. Two puddles with different Bond numbers demonstrate the different drop shapes resulting from varying body forces or surface tension. These nondimensionalized shapes can be scaled up or down to fit an empirical drop.

A physical drop can be matched against the family of shapes to determine the best fit, and thereby infer the Bond number for that liquid-gas interface. This match can be effectively determined by a computer program [4], or more simply by inspection.

B. Contact Angle

The puddle shape defined by the Bond number is independent of the adhesive forces between the solid and the liquid, and in fact describes a drop in which the two phases do not interact. The boundary of the liquid then lies tangent to the solid surface. Any adhesion between the two will cause the liquid-solid interface to have non-zero extent. This results in the tangent to the liquid-air interface intersecting the solid surface. The angle between the two through the liquid volume is defined to be the contact angle θ_C. This is shown in figure 3.

The drop geometry described by the Bond number is truncated by the plane of the solid to give the final shape of the puddle; the resulting contact angle of the solid-liquid interface is defined by the adhesion energy ΔW of the liquid per unit area to

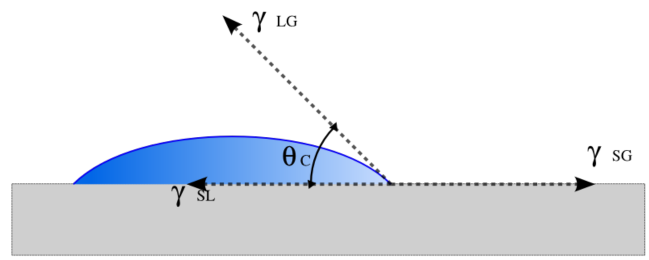

Fig. 3. The contact angle θ_C of a liquid drop on a solid surface is a function of the adhesive forces between the two phases. Image from [5].

the solid in the gaseous medium. They are related by the Young-Dupre equation [5]:

$$\sigma(1 + \cos(\theta_C)) = \Delta W. \quad (2)$$

Horizontal surfaces intersecting a drop at varying contact angles and the resulting puddle shapes are shown in figure 4.

III. Kitty-Puddle Fitting

The profile of the puddle shown in 1 can now be analyzed to characterize the feline fluid. The contact angle is measured to average 130^o between the advancing and receding edges of the drop. Fitting a drop shape cut off at $\theta_{C_{kitty}} = 130^o$ around the cat, the Bond number can be approximated by $Bo_{kitty} = 1$. The resulting theoretical puddle is overlaid on the empirical photograph in figure 5.

The kitten doesn't fit completely inside the predicted shape; the posterior of the animal protrudes beyond the expected boundary. This is a manifestation of the non-ideality of the surface on which the kitty-puddle rests. Due to the macro- and microscopic irregularities in surface texture, there are a range of stable interfacial states, and a dynamic analysis of the formation of the drop would be necessary to account for the deviation from the ideal drop shape [6]. However, because the bulk of the fluid fits the predicted curve quite well, we can ignore the slight perturbation in our consideration of bulk kitten fluid parameters.

A. Physical kitten parameters

According to [7], the average weight of a Moggy cross breed small cat is 3.4kg. Modeling the cat as

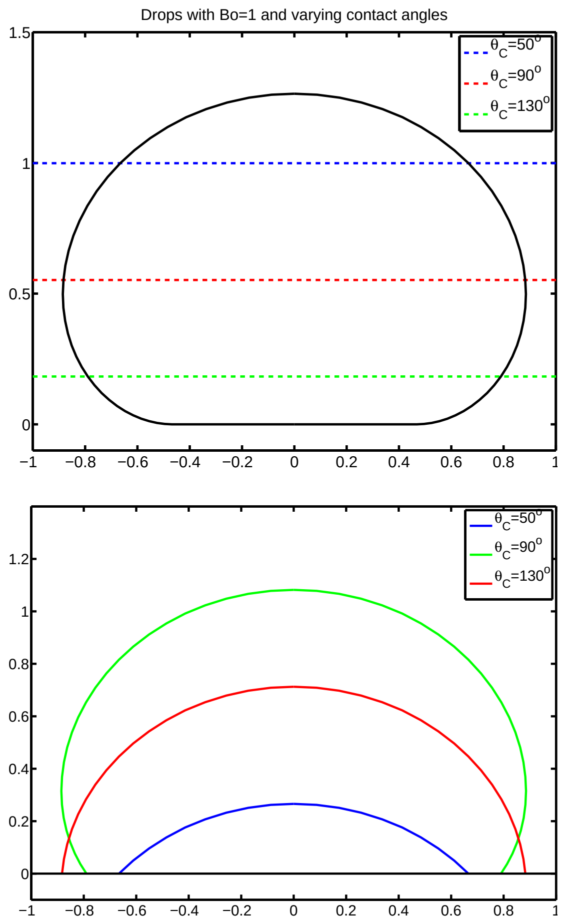

Fig. 4. Varying surface adhesion energy causes the base of a drop to spread out along a surface. For a given bond number, this has the effect of truncating the base of the drop shape at a given contact angle.

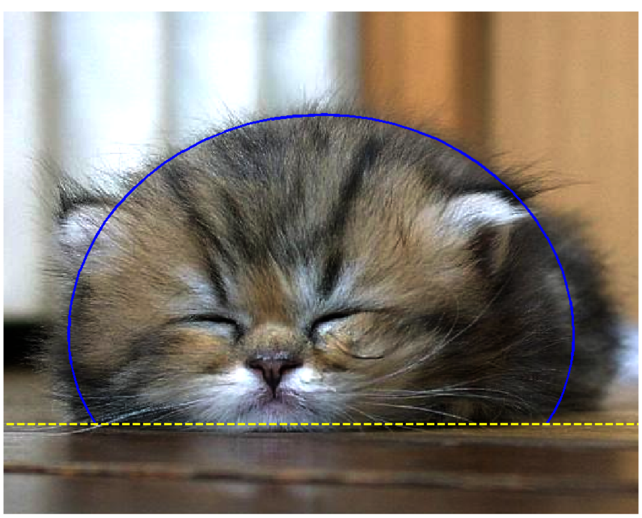

Fig. 5. Cat in a bubble!

a cylinder of radius $6cm$ and length $35cm$ yields a density

$$\rho_{kitty} = \frac{3.4kg}{\pi(.06m)^2(.35m)} = 859kg/m^3.$$

B. *Surface tension of a kitten*

Equation (1) for the Bond number can be inverted to calculate the surface tension of the kitten:

$$\sigma_{kitty} = \frac{gr_0^2(\rho_{kitty} - \rho_{air})}{Bo}. \tag{3}$$

Fitting in parameters described above gives

$$\begin{aligned} \sigma_{kitty} &= \frac{9.81\frac{m}{s^2}(0.06m)^2(859\frac{kg}{m^3} - 1.2\frac{kg}{m^3})}{1} \\ &= 30N/m. \end{aligned}$$

C. *Adhesion energy of a kitten on a hardwood floor*

The Young-Dupre equation (2) can now be used to calculate the adhesion energy per unit area of the cat to a hardwood floor in an air medium. Given the observed contact angle and calculated surface tension,

$$\begin{aligned} \Delta W &= \sigma_{kitty}(1 + \cos(\theta_{C_{kitty}}) \\ &= 10.7J/m^2. \end{aligned}$$

The estimated (from literature), measured (from the image), and calculated parameters for the representative feline sample are summarized in table I.

TABLE I
FELINE FLUID PARAMETER VALUES

Parameter	Description	Type	Value	Units
r_0	Radius of a kitten	E	6	cm
g	Acceleration (gravity)	E	9.81	m/s^2
ρ_{kitty}	Density of a kitten	C	859	kg/m^3
ρ_{air}	Density of air	E	1.2	kg/m^3
Bo_{kitty}	Bond number	M	1	
σ_{kitty}	Surface tension	C	30	N/m
$\theta_{C_{kitty}}$	Contact angle	M	130^o	
ΔW	Adhesion Energy	C	10.7	J/m^2

IV. CONCLUSION

This work features the first foray into the field of fauna fluids, focusing on felines.

The calculated surface tension of the kitten is over two orders of magnitude greater than that of water at room temperature $\sigma = 0.0728 N/m$, and still notably higher than mercury $\sigma = .4254 N/m$ [8]. This indicates great promise for the use of kittens as a new class of heat transfer fluid.

The high contact angle of the kitten on a hardwood floor is indicative of a strongly ailurophobic surface. In fact, the rather low interfacial adhesion energy is less than the energy required to lift the cat just $2cm$. This explains the ease at which napping kitty-puddles can be picked off of the floor.

Fig. 6. Further examples of fluid puddles from throughout the animal kingdom. Top row: bunny [9], hedgehog [10]. Bottom row: penguin [11], baby [12].

Kittens aren't the only animals that form drops on solid surfaces. It would be interesting to conduct further research into establishing fluid properties for other animals to compare with kittens and other conventional liquids. Some examples of animal puddles are shown in figure 6.

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