Optimal Scheduling for Resource-Constrained Multirobot Cooperative Localization

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***Abstract*—Typically, localization algorithms focus on maximizing estimation accuracy, often while neglecting some of the real-world costs of achieving such performance. In this letter, we formulate *optimal scheduling problems* of multirobot localization to investigate the tradeoff between estimate accuracy and its corresponding cost. We derive the continuous-time approximation to explicitly represent the rates of distinct operations in a multirobot localization scheme, which we then use to define the optimization problems for scheduling those operations. For a given accuracy requirement, we can solve the *cost minimization problem* to find the lowest cost schedule satisfying the prescribed error expectation; similarly, for a given cost budget, we can solve the *trace minimization problem* to find a schedule for the best achievable error performance. While the interplay between the localization error and the operation parameters is described by a Riccati equation, the Fréchet derivative is applied to shed the light on their implicit relationship. In a localization example grounded in a realistic setting, we show that by solving these optimal scheduling problems, power consumption can be reduced by 64% through cost minimization, or the bounding covariance trace can be reduced by 15% through trace minimization.**

***Index Terms*—Localization, multi-robot systems, distributed robot systems, optimization and optimal control.**

I. Introduction

LOCALIZATION is essential to the successful deployment of multirobot systems. Extensive research has been conducted on algorithm development in order to maximize localization accuracy, but the analysis of the costs required to achieve such localization performance has mostly been neglected. In general, physical resource constraints, including power, energy, and processor time, impose costs on constituent operations of an algorithm. Thus, one must evaluate localization algorithms in the context of constrained resources to complete their characterization. In this work, we analyze and characterize the tradeoff between error performance and the associated costs of executing a localization algorithm.

Localization in multirobot systems can be achieved cooperatively, which relies on several operations including proprioceptive odometry input, exteroceptive observation, and communication of information between robots. While most of the existing cooperative localization algorithms require communication directly after observation, considering the two steps as a single joint operation, a cooperative localization algorithm that decouples each operation into independent information sources while preserving the estimation consistency was proposed in [1]. With observation and communication operations separated, we can now explicitly analyze the costs associated with each operation in a direct way. In this work, we show how to optimize the localization performance against overall operation cost by independently varying the frequency of each operation.

The main parameters of this cooperative localization algorithm include the operation rates and the covariance intersection (CI) parameter α in the communication update step [2]. For operation rates, odometry propagation rate f_p, observation rate f_o, and communication rate f_c are considered for each operation respectively. While higher operation rate requires higher associated operation cost, we can regard these operation rates as a tradeoff between the localization performance and the cost to achieve such accuracy. Formally, the optimal scheduling problem is to find the optimal values of f_p, f_o, f_c and α that minimize either the total operation cost or the localization uncertainty within the system constraints. We explicitly formulate the cost minimization problem and the trace minimization problem covering these two cases.

While operation rates are critical in optimal scheduling problems, the discrete-time modeling is vulnerable to the change of operation rates. In the localization algorithm, each operation occurs at a single time instance, and the system is naturally analyzed as a sequence of discrete operations. However, any tractable analysis in discrete-time modeling strictly requires the existence of recurrent points whose interval has integer number of operations. To explicitly represent the operation rates, we instead approximate the original discrete sequence by a limiting continuous-time system. The corresponding covariance update then follows a continuous-time Riccati recursion, and the steady-state localization uncertainty can be represented by the solution of the corresponding continuous-time algebraic Riccati equation (CARE).

By relating the operation rates and the localization accuracy with the CARE, the optimal scheduling problem can then be formulated as an optimization problem. Before solving the optimization problem explicitly, we delineate the effect of operation rates on the localization accuracy. In particular, since the communication update involves CI, which is a fusion scheme without the knowledge of correlation but introducing additional uncertainty, the effect of communication rate f_c on the localization accuracy is less obvious, and will be discussed in detail by the Fréchet derivative in this letter.

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The main contributions of this paper include:

- the formal derivation of the limiting CARE that characterizes the operation rates f_p, f_o, f_c from the original discrete-time algebraic Riccati recursion,
- the formulation of the *cost minimization problem* and the *trace minimization problem* for the optimization of the operation rates,
- the relationship between the stationary covariance Π and the operation rates based on Fréchet derivative, and
- an explanatory example simulated from a real-world grounding demonstrating the cost savings and improved accuracy resulting from optimal scheduling.

The organization of this paper is as follow: The related work is reviewed in the next section. The system model together with the localization algorithm is presented in Section III. In Section IV, the procedure leading to continuous-time model is explicated, with the following section devoted to the properties of the derived CARE. The formulation of the optimal scheduling problem is offered in Section VI. Section VII illustrates the effectiveness of the optimal scheduling problem with an explanatory example. Finally, the conclusion of this paper is addressed in Section VIII.

II. Related Work

To the best of our knowledge, the first and the only work on the optimal scheduling problem in robotic localization algorithm is in [3], based on the benchmark extended Kalman filter (EKF) multirobot localization algorithm [4]. The goal in [3] is to minimize the estimation uncertainty given different sensor characteristics. With this former work as basis, there are 3 main improvements in this letter: 1) Since all the robots should update their estimates after the observation update in [4], the resource constraint in their formulation implicitly includes the communication cost as well. In contrast, our underlying algorithm already separates observation and communication. Therefore, we can explicitly present the communication cost, and optimize the overall performance. 2) We use a general tool, the Fréchet derivative, to investigate the effect of operation rates in the CARE. The demonstration of the analysis procedure not only explains the effect of the communication related parameters but can also be generalized in arbitrary CAREs. 3) The scenario in [3] requires that robots to stay in fixed formation, which limits the applicability of the conclusion. We consider a localization scenario without imposing any formation constraint, and consider the upper bound of the estimation covariance instead. Therefore, our results are applicable in various robotic systems conducting localization algorithms.

While the operation cost is not explicitly considered in the works following the benchmark localization algorithm [4], some work do serve to implicitly minimize such cost, e.g. by reducing the communication overhead. In those works, one approach preserves the centralized-equivalent EKF modeling, and reduces communication by carefully designing the exchanged information [5] or by introducing a central unit [6] in charge of tracking the correlation. The other approach, including [1], [7], [8], uses the information fusion method that does not require the full correlation information, and thus needs less communication in the algorithm. However, the aforementioned works do not comprehensively consider the tradeoff between cost and performance, which leaves the characterization of localization algorithm incomplete. In this letter, we extend the localization algorithm in [1], varying operation rates independently for the optimization problems.

In sensor networks and general estimation schemes, the discussion on operation scheduling is extensive. Among all scheduling problems, we focus on the scheduling of information availability due to its direct relation to this work. In sensor networks, one of the mainstream approaches is to trigger communication by certain event, rather that by some predetermined procedure, which decreases the overall cost [9], and this scheme is already applied in distributive estimation algorithm, including CI [10] and consensus KF [11]. Another direction of investigation confronts the stochastic nature of information availability, either by predetermined sensor selection algorithms [12], or by the unreliable communication channel [13]–[15]. Indeed, the event-triggering scheme, the expected evolution of stochastic modeling and the constant rate scheduling of this paper all lead to similar mathematical formulation, which suggests the underlying commonality among three schemes for further investigation. However, while all other works discuss with the scheduling of raw observation information, this paper takes one step further to incorporate estimation exchange as well.

III. System Model and the Localization Algorithm

This section is a summary of [1] for self-consistency. In the following, the foundation of the localization algorithm is presented, together with the upper bound on covariance update with constant coefficients, which is used in constructing the Riccati recursion later. We consider a 2D multirobot system where robots are indexed by $\{1, \ldots, N\}$, and several landmarks whose locations are known by the robots in advance. The position of robot i at time t is regarded as the state, denoted as $s_{i,t} = [x_{i,t}, y_{i,t}]^\mathsf{T}$ The state of the whole system is denoted by $s_t = [s_{1,t}^\mathsf{T}, \ldots, s_{N,t}^\mathsf{T}]^\mathsf{T}$.

A. System Model

The motion model describes the spatial displacement of robots due to odometry inputs. While the framework is not limited to any specific models, we mainly consider the velocity input $v_{i,t}$ in this letter. Let T_p be the interval between two motion propagation updates. The state of robot i at the next time is then given by

$$s_{i,t+1} = f_i(s_{i,t}, v_{i,t}) = \begin{bmatrix} x_{i,t} + v_{i,t} T_p \cos(\theta_{i,t}) \\ y_{i,t} + v_{i,t} T_p \sin(\theta_{i,t}) \end{bmatrix}, \quad (1)$$

where $\theta_{i,t}$ is the orientation of robot i at time t.

As for the observation model, if robot i observes an object j, the relative position obtained by robot i is

$$o_{ij} = C^\mathsf{T}(\theta_{i,t}) \left(\begin{bmatrix} x_{j,t} \\ y_{j,t} \end{bmatrix} - \begin{bmatrix} x_{i,t} \\ y_{i,t} \end{bmatrix} \right) = C^\mathsf{T}(\theta_{i,t}) H_i s_t, \quad (2)$$

where $C(\theta)$ is the rotation matrix with argument θ. Explicitly, if the object j is a landmark,

$$H_i = \begin{bmatrix} 0_{2\times 2} & \cdots & \underbrace{-I_2}_{i} & \cdots & 0_{2\times 2} \end{bmatrix},$$

where I_n is the $n \times n$ identity matrix. Otherwise, if the object j is robot j,

$$H_i = \begin{bmatrix} 0_{2\times 2} & \cdots & \underbrace{-I_2}_{i} & \cdots & \underbrace{I_2}_{j} & \cdots & 0_{2\times 2} \end{bmatrix}.$$

In this letter, we consider that the observation is accomplished by distance and bearing sensors, and the relative position can then also be expressed as

$$o_{ij} = d_{ij} \begin{bmatrix} \cos(\phi_{ij}) \\ \sin(\phi_{ij}) \end{bmatrix}, \tag{3}$$

based on the relative distance d_{ij} and relative bearing ϕ_{ij}.

B. Localization Algorithm

The localization algorithm is based on EKF and CI, where each robot i keeps an estimate of s_t, denoted by $\hat{s}_t^i$, together with its covariance $\Sigma_{i,t}$.

1) Motion Propagation Update: We consider robot i for example, and the covariance update of motion propagation is

$$\Sigma_{i,t+1} = \Sigma_{i,t} + T_p^2 Q_u, \tag{4}$$

where Q_u is determined by the input noise at each robots. Concerning the fact that all the noises at each robot are independent,

$$Q_u = \mathsf{Diag}(Q_{u_1}, \ldots, Q_{u_N}).$$

The exact value of Q_{u_i} depends on the availability of the input information. Since the input v_i is available at robot i,

$$Q_{u_i} = \begin{bmatrix} \cos(\theta_{i,t}) \\ \sin(\theta_{i,t}) \end{bmatrix} \sigma_{\mathbf{n}_v}^2 \begin{bmatrix} \cos(\theta_{i,t}) & \sin(\theta_{i,t}) \end{bmatrix},$$

where $\sigma_{\mathbf{n}_v}^2$ is the variance of the velocity input noise. For $j \neq i$, the velocity inputs v_j are no longer available for robot i. Without the exact value, the input itself is modeled as a Gaussian random variable $\mathbf{v}_j$ with variance $\sigma_{\mathbf{v}}^2$. The covariance increment is now given by

$$Q_{u_j} = \sigma_{\mathbf{v}}^2 I_2, \quad j \neq i. \tag{5}$$

2) Observation Update: The observation step updates the estimate based on the exteroceptive measurements o_{ij}, which is standard in EKF procedure. Based on (2), the innovation $\tilde{o}_{ij} = o_{ij} - \hat{o}_{ij}$ can be approximated as

$$\tilde{o}_{ij} \approx C^{\mathsf{T}}(\theta_{i,t}) H_i \tilde{s}_{t^-}^i + \mathbf{n}_{o_{ij}}.$$

Such approximation distinguishes two uncorrelated error terms: the estimation error $\tilde{s}_{t^-}^i$, and the measurement noise $\mathbf{n}_{o_{ij}}$. While the observation result o_{ij} is obtained by distance and bearing sensors, the covariance matrix $R_{o_{ij}}$ of $\mathbf{n}_{o_{ij}}$ can be expressed as

$$R_{o_{ij}} = C(\phi_{ij}) \mathsf{diag}(\sigma_{d_i}^2, d_{ij}^2 \sigma_{\phi_i}^2) C^{\mathsf{T}}(\phi_{ij}).$$

The overall covariance update can be expressed as

$$\Sigma_{i,t^+}^{-1} = \Sigma_{i,t^-}^{-1} + H_i^{\mathsf{T}} C(\theta_{i,t}) R_{o_{ij}}^{-1} C^{\mathsf{T}}(\theta_{i,t}) H_i. \tag{6}$$

3) Communication Update: In communication update, robot i updates its estimate information with the external information from other robots by CI. In this letter, we only consider that only one robot k sends information to robot i. The generalization to arbitrary number of robots is direct and does not alter the framework presented in the following. Explicitly, the covariance update is given by

$$\Sigma_{i,t^+}^{-1} = \alpha \Sigma_{i,t^-}^{-1} + (1-\alpha) \Sigma_k^{-1}, \quad \alpha \in (0,1). \tag{7}$$

With larger α, the estimation in robot i itself is more trusted in the estimation fusion.

C. Upper Bound of the Covariance Update

We mainly focus on the covariance update in rest of the paper, (4), (6), and (7) in particular, since it characterizes the estimation uncertainty of the localization algorithm. However, the coefficients in those equations are state dependent, which obscures the tractability of our targeted parameters. We instead find the update equations $\Pi_{i,t}$ such that the coefficients are independent of state as in [16], and the updated covariances are no smaller than those updated by the corresponding original equations.

For propagation update, only Q_{u_i} is state-dependent. Since $Q_{u_i} \leq \sigma_{\mathbf{n}_v}^2 I_2$, we set

$$\Pi_{i,t+1} = \Pi_{i,t+1} + T_p^2 Q, \tag{8}$$

with

$$Q = \mathsf{Diag}(Q_{u_1}, \ldots, Q_{u_{i-1}}, \sigma_{\mathbf{n}_v}^2 I_2, Q_{u_{i+1}}, \ldots, Q_{u_N}).$$

For observation update, we take

$$R_i = \max(\sigma_{d_i}^2, d_{\max}^2 \sigma_{\phi_i}^2) I_2 \geq R_{o_{ij}}, \tag{9}$$

with $d_{\max}$ the maximum observed distance. We then replace the original update (6) with

$$\Pi_{i,t^+}^{-1} = \Pi_{i,t^-}^{-1} + H_i^{\mathsf{T}} R_i^{-1} H_i \tag{10}$$

The constant update equation is not only independent of the state but also of the observation result.

As for communication update, since the covariance evolution depends on the input information, we set two assumptions on the information provider k for presentation clarity. 1) Robot k does not receive other communication input. 2) The observation rate of robot j is larger the communication rate of robot i. With these two assumptions, we have the state covariance sent to robot i from robot k, denoted by Σ_{k,t^*}, satisfies the inequality:

$$\begin{aligned} \Sigma_{k,t^*}^{-1} &\geq \Sigma_{k,t^-}^{-1} + H_k^{\mathsf{T}} C(\theta_k) R_{o_{kl}}^{-1} C^{\mathsf{T}}(\theta_k) H_k \\ &\geq H_k^{\mathsf{T}} C(\theta_k) R_{o_{kl}}^{-1} C^{\mathsf{T}}(\theta_k) H_k \\ &\geq H_k^{\mathsf{T}} R_k^{-1} H_k, \end{aligned}$$

where R_k is defined similarly in (9). The two assumptions applied here are only for derivation simplicity, and do not change the framework in the following sections. As long as the topology and the rate at robot k are fixed, the constant upper bound of Σ_{k,t^*} with the two assumptions relaxed can still be obtained . Therefore, we have

$$\Pi_{i,t^+}^{-1} = \alpha \Pi_{i,t^-}^{-1} + (1-\alpha) H_k^{\mathsf{T}} R_k^{-1} H_k. \tag{11}$$

To sum up, the covariance update equations of $\Sigma_{i,t}$ in the localization algorithm, described by (4), (6) and (7), are state

Algorithm 1: The Exact and the Upper-Bound Covariance Updates of the Multirobot Cooperative Localization Algorithm.

Motion Propagation

$$\Sigma_{i,t+1} = \Sigma_{i,t} + T_p^2 Q_u,$$
$$\Pi_{i,t+1} = \Pi_{i,t+1} + T_p^2 Q.$$

Observation

$$\Sigma_{i,t^+}^{-1} = \Sigma_{i,t^-}^{-1} + H_i^\top C(\theta_{i,t}) R_{o_{ij}}^{-1} C^\top(\theta_{i,t}) H_i,$$
$$\Pi_{i,t^+}^{-1} = \Pi_{i,t^-}^{-1} + H_i^\top R_i^{-1} H_i.$$

Communication

$$\Sigma_{i,t^+}^{-1} = \alpha \Sigma_{i,t^-}^{-1} + (1-\alpha)\Sigma_k^{-1},$$
$$\Pi_{i,t^+}^{-1} = \alpha \Pi_{i,t^-}^{-1} + (1-\alpha) H_k^\top R_k^{-1} H_k.$$

dependent. We instead find the constant update equations of $\Pi_{i,t}$ by (8), (10) and (11) for respect updates. Both covariance updates are summarized in Algorithm 1. With the same initial covariance, or $\Sigma_{i,0} = \Pi_{i,0}$, we have

$$\Sigma_{i,t} \leq \Pi_{i,t}$$

for all t. Therefore, we interpret the constant update equations as the upper bound of the exact covariance, and optimize over the upper bound instead.

IV. Continuous-Time Riccati Recursion With Operation Rates

A. Discrete-Time Riccati Recursion

To obtain the limiting continuous-time Riccati recursion, we have to construct the discrete-time modeling concerning the three types of updates. With operations being taken at different rates, we first define a time interval of the discrete time model T equal to the period of communication update T_c. This implies that there are T_c/T_p motion propagation updates, T_c/T_o observation updates, and 1 communication update in each interval T. Even though the numbers of each action are fixed, the covariance update is still undetermined since the specific order of those actions is unspecified. We instead approximate the covariance update by reordering the operations: first observations, then communication, and then motion propagation.

In each time interval, the covariance update after observation and communication updates can be expressed as

$$\begin{aligned}\Pi_{t^*} &= \left[\alpha \Pi_t^{-1} + \alpha \frac{T_c}{T_o} H_i^\top R_i^{-1} H_i + (1-\alpha) H_k^\top R_k^{-1} H_k\right]^{-1} \\ &= \left[\alpha \Pi_t^{-1} + \alpha \check{H}^\top \check{R}^{-1} \check{H}\right]^{-1} \\ &= \frac{1}{\alpha}\Pi_t - \frac{1}{\alpha}\Pi_t \check{H}^\top \left[\frac{1}{\alpha}\check{R} + \frac{1}{\alpha}\check{H}\Pi_t\check{H}^\top\right]^{-1} \check{H}\frac{1}{\alpha}\Pi_t \\ &= \frac{1}{\alpha}\Pi_t - \frac{1}{\sqrt{\alpha}}\Pi_t \check{H}^\top \left(\check{R} + \check{H}\Pi_t\check{H}^\top\right)^{-1} \check{H}\frac{1}{\sqrt{\alpha}}\Pi_t, \end{aligned} \quad (12)$$

where

$$\check{H} = \begin{bmatrix} H_i \\ H_k \end{bmatrix}, \quad \check{R} = \begin{bmatrix} \frac{T_o}{T_c} R_i & \\ & \frac{\alpha}{1-\alpha} R_k \end{bmatrix}.$$

The exact motion propagation update in a single interval is given by

$$\Pi_{i,t+1} = \Pi_{i,t^*} + \frac{T_c}{T_p}\left(\frac{T_p}{T_c}T\right)^2 Q = \Pi_{i,t^*} + \frac{T_p}{T_c} T^2 Q. \quad (13)$$

Combining (12) and (13), the overall covariance update can be obtained as

$$\begin{aligned}\Pi_{t+1} &= \check{F}\Pi_t\check{F}^\top + \frac{T_p}{T_c}T^2 Q \\ &\quad - \check{F}\Pi_t\check{H}^\top\left(\check{R} + \check{H}\Pi_t\check{H}^\top\right)^{-1}\check{H}\Pi_t\check{F}^\top, \end{aligned} \quad (14)$$

with $\check{F} = \frac{1}{\sqrt{\alpha}} I$. This overall covariance update is exactly a discrete-time Riccati recursion.

B. Limiting Continuous-Time Riccati Recursion

From the discrete-time Riccati recursion (14), the corresponding limiting continuous-time Riccati recursion can be obtained with the procedure in [17]. To begin with, we let $\check{F}_c = \frac{1}{T}(\check{F} - I)$, and rewrite the update equation as

$$\begin{aligned}\frac{\Pi_{t+1} - \Pi_t}{T} &= \check{F}_c\Pi_t + \Pi_t\check{F}_c^\top + \frac{T_p}{T_c}TQ \\ &\quad - \Pi_t\check{H}^\top\left(T\check{R} + T\check{H}\Pi_t\check{H}^\top\right)^{-1}\check{H}\Pi_t + \frac{o(T)}{T}.\end{aligned}$$

By taking $T \to 0$ while keeping $\check{F}_c$, $T\check{R}$, and TQ constant, we obtain the limiting continuous-time Riccati recusion

$$\dot{\Pi}(t) = \check{F}_c\Pi(t) + \Pi(t)\check{F}_c^\top - \Pi(t)\check{H}^\top\check{R}_c^{-1}\check{H}\Pi(t) + Q_c. \quad (15)$$

Since $T\check{R}$ and TQ are constant, we can determine $\check{R}_c$ and Q_c with their initial values. That is,

$$\check{R}_c = T_c\check{R} = \frac{1}{f_c}\check{R} = \begin{bmatrix} \frac{1}{f_o}R_i & \\ & \frac{1}{f_c}\frac{\alpha}{1-\alpha}R_k \end{bmatrix}, \quad (16)$$

and

$$Q_c = \frac{T_p}{T_c}T_c Q = \frac{1}{f_p}Q. \quad (17)$$

The order in the original discrete-time recursion (12) does matter in the limiting continuous-time approximation. For example, if the order of communication and observation operations are interchanged, the resulting continuous-time recursion differs. While this effect is negligible in the following optimization, detailed analysis should be further explored.

V. Properties of the CARE

A. Convergence of the CARE

After establishing the relationship between the original discrete-time and the limiting continuous-time processes, we further require that the limiting continuous-time process inherits the convergence property from the original discrete-time counterpart. The convergence of estimation covariance in discrete-time modeling, which mainly depends on the observation and communication topologies, is discussed thoroughly in [1], and we only consider the convergent case in this letter to emphasize the scheduling aspect.

Theorem 1: If $(\check{F}, Q^{\frac{1}{2}})$ is stabilizable and $(\check{F}, H)$ is detectable in the discrete-time model, then both the discrete-time Riccati recursion (14) and the associated continuous-time Riccati recursion (15) converge to the solutions of the corresponding Riccati equations with positive semidefinite initial condition, respectively.

Proof: The convergence of (14) is direct by [18]. Since the eigenvalues of $\check{F}$ are all $\frac{1}{\sqrt{\alpha}} > 1$, every mode is controllable, or the controllability matrix

$$\mathcal{C} = [Q^{\frac{1}{2}}, \check{F}Q^{\frac{1}{2}}, \ldots, \check{F}^{M-1}Q^{\frac{1}{2}}]$$

is full rank, with $M = 2N$. The controllability matrix $\mathcal{C}_c$ of $(\check{F}_c, Q^{\frac{1}{2}})$ differs from $\mathcal{C}$ only by a constant multiplication, and is full rank as well. Therefore, the pair $(\check{F}_c, Q^{\frac{1}{2}})$ is controllable, thus stabilizable. The detactability of $(\check{F}_c, H)$ can be proved similarly. Jointly, the convergence of the continuous-time Riccati recursion with positive semidefinite initial condition follows [18].

Even though the conditions in Theorem 1 is only sufficient, they are very close to necessary condition with some tradeoff between initial condition and the requirement on controllability and observability. The details can be found in [18]. ∎

With Theorem 1, once we assure the convergence in the discrete-time scenario, the convergence of the continuous-time recursion (15) is guaranteed, and the recursion converges to the solution Π of the corresponding CARE

$$\check{F}_c\Pi + \Pi\check{F}_c^{\mathsf{T}} - \Pi\check{H}^{\mathsf{T}}\check{R}_c^{-1}\check{H}\Pi + Q_c = 0. \tag{18}$$

The solution Π can then be interpreted as the estimation uncertainty of the particular configuration of the operable parameters, f_p, f_o, f_c and CI parameter α.

B. Effects of Parameters on Π

Even though (18) can be solved numerically without difficulty, the analytical effects of those parameters on Π are still of great engineering concern. As all operations are associated with certain costs, one of the practical concern involves how should one allocate the operation resource to decrease the uncertainty Π in the most efficient way. It can be shown that Π decreases when increasing f_p or f_o based on the result in [19]. However, the effects of f_c and α on Π is less obvious. Conceptually, we want to know the implicit derivative of Π with respect to f_c and α in (18), which can be explicated by the Fréchet derivative in functional analysis [20].

To begin with, the recursion (15) is rewritten as a function of operation parameters

$$G(\Pi, f_c, \alpha) = 2f_c\left(\frac{1}{\sqrt{\alpha}} - 1\right)\Pi - \Pi\check{H}^{\mathsf{T}}\check{R}_c^{-1}(f_c, \alpha)\check{H}\Pi + Q_c, \tag{19}$$

and the CARE (18) with solution $(\Pi_0, f_{c,0}, \alpha_0)$ is exactly

$$G(\Pi_0, f_{c,0}, \alpha_0) = 0. \tag{20}$$

1) The Communication Rate: By applying the implicit function theorem of the Fréchet derivative on (20),

$$G_{f_c}\Delta f_c + G_\Pi \Pi_{f_c}\Delta f_c = 0.$$

With some manipulation, the Fréchet derivative of $\Pi_{f_c}\Delta f_c$ at $(\Pi_0, f_{c,0}, \alpha_0)$ is given by

$$\Pi_{f_c}\Delta f_c = -G_\Pi^{-1}G_{f_c}\Delta f_c. \tag{21}$$

For the first operator, $G_{f_c}\Delta f_c$ is the Fréchet derivative at $(\Pi_0, f_{c,0}, \alpha_0)$, given by

$$G_{f_c}\Delta f_c = 2\left(\frac{1}{\sqrt{\alpha_0}} - 1\right)\Pi_0\Delta f_c + G_{\check{R}_c}\check{R}_{c,f_c}\Delta f_c,$$

where

$$G_{\check{R}_c}\Delta\check{R}_c = \Pi_0\check{H}^{\mathsf{T}}\check{R}_{c,0}^{-1}\Delta\check{R}_c\check{R}_{c,0}^{-1}\check{H}\Pi_0,$$

with $\check{R}_{c,0} = \check{R}_c(f_{c,0}, \alpha_0)$ and

$$\check{R}_{c,f_c}\Delta f_c = \begin{bmatrix} 0 & \\ & -\frac{1}{f_{c,0}^2}\frac{\alpha_0}{1-\alpha_0}R_k \end{bmatrix}\Delta f_c.$$

In the following, the operator $G_\Pi^{-1}\Delta\Pi$ is the inverse of the operator

$$G_\Pi\Delta\Pi = F_{\text{cl},0}\Delta\Pi + \Delta\Pi F_{\text{cl},0}^{\mathsf{T}},$$

and

$$F_{\text{cl},0} = \check{F}_c(\alpha_0) - \Pi_0\check{H}^{\mathsf{T}}\check{R}_{c,0}^{-1}\check{H}.$$

Therefore, the output of the operator $G_\Pi^{-1}\Delta\Pi$ is the solution X of the Lyapunov equation

$$F_{\text{cl},0}X + XF_{\text{cl},0}^{\mathsf{T}} = \Delta\Pi.$$

To sum up, the output of the operator $\Pi_{f_c}\Delta f_c$ is the solution X of the Lyapunov equation

$$\begin{aligned} F_{\text{cl},0}X + XF_{\text{cl},0}^{\mathsf{T}} &= -G_{f_c}\Delta f_c \\ &= -2\left(\frac{1}{\sqrt{\alpha_0}} - 1\right)\Pi_0\Delta f_c - G_{\check{R}_c}\check{R}_{c,f_c}\Delta f_c. \end{aligned} \tag{22}$$

To interpret the derivative $\Pi_{f_c}\Delta f_c$, we let the solution $X = X_1 + X_2$ in (22), where X_1 is the solution of

$$F_{\text{cl},0}X_1 + X_1F_{\text{cl},0}^{\mathsf{T}} = -2\left(\frac{1}{\sqrt{\alpha_0}} - 1\right)\Pi_0\Delta f_c$$

and X_2 is the solution of

$$F_{\text{cl},0}X_2 + X_2F_{\text{cl},0}^{\mathsf{T}} = -G_{\check{R}_c}\check{R}_{c,f_c}\Delta f_c.$$

For $\Delta f_c \geq 0$, or by increasing communication rate f_c,

$$-2\left(\frac{1}{\sqrt{\alpha_0}} - 1\right)\Pi_0\Delta f_c \leq 0,$$

and

$$-G_{\check{R}_c}\check{R}_{c,f_c}\Delta f_c \geq 0.$$

Since $(\check{F}_c, Q^{\frac{1}{2}})$ is stabilizable and $(\check{F}_c, H)$ is detectable, $F_{\text{cl},0}$ is stable [18]. Therefore, as the solution of Lyapunov equation, we have $X_1 \geq 0$ and $X_2 \leq 0$. That is, by increasing the communication rate f_c, the increasing part of Π is characterized by X_1 with the effect of $\check{F}_c$, and the decreasing part is given by X_2 with the effect of R_k. In other words, the communication update provides the information from $H_k^\mathsf{T} R_k^{-1} H_k$ with the cost that the original covariance is enlarged at the same time. This analysis not only substantiates the qualitative understanding, but also characterizes quantitative effect as well.

2) The CI Coefficient: The same analysis applied on the CI coefficient is direct. At $(\Pi_0, f_{c,0}, \alpha_0)$,

$$\Pi_\alpha \Delta\alpha = -G_\Pi^{-1} G_\alpha \Delta\alpha, \tag{23}$$

with

$$G_\alpha \Delta\alpha = -f_{c,0}\alpha_0^{-\frac{3}{2}} \Pi_0 \Delta\alpha + G_{\check{R}_c} \check{R}_{c,\alpha} \Delta\alpha,$$

and

$$\check{R}_{c,\alpha}(f_{c,0}, \alpha_0)\Delta\alpha = \begin{bmatrix} 0 & \\ & \dfrac{1-\alpha_0+\alpha_0^2}{f_{c,0}(1-\alpha_0)^2} R_k \end{bmatrix} \Delta f_c.$$

Similarly, we can write $\Pi_\alpha \Delta\alpha = Y_1 + Y_2$ with

$$F_{\text{cl},0} Y_1 + Y_1 F_{\text{cl},0}^\mathsf{T} = f_{c,0}\alpha_0^{-\frac{3}{2}} \Pi_0 \Delta\alpha$$

and

$$F_{\text{cl},0} Y_2 + Y_2 F_{\text{cl},0}^\mathsf{T} = -G_{\check{R}_c} \check{R}_{c,\alpha} \Delta\alpha.$$

With $\Delta\alpha \geq 0$, we have $Y_1 \leq 0$ and $Y_2 \geq 0$. Increasing α represents emphasizing the internal estimate over received data at the communication step, which decreases Π with the effect of $\check{F}_c$ but increases Π by that of $H_k^\mathsf{T} R_k^{-1} H_k$.

VI. Optimal Scheduling Problems

To investigate localization accuracy as well as the associated costs, we formulate two optimization problems: to minimize the cost with predetermined covariance constraints, or to minimize the covariance with a resource limit. In the localization algorithm, one can design the observation rate f_o and the communication rate f_c, as well as the CI parameter α. Though analysis similarly extends to the propagation rate f_p, we do not include it in the optimization variables since it is generally determined by the underlying odometry configuration.

A. Cost Minimization Problem

We first consider the optimal scheduling problem that aims to minimize the overall cost of the localization algorithm, while maintaining a specified accuracy requirement.

$$\begin{aligned} \underset{f_o, f_c, \alpha}{\text{minimize}} \quad & \mu_o f_o + \mu_c f_c \\ \text{subject to} \quad & 2 f_c \left(\frac{1}{\sqrt{\alpha}} - 1 \right) \Pi + Q_c \\ & \quad - \Pi \check{H}^\mathsf{T} \check{R}_c^{-1}(f_o, f_c, \alpha) \check{H} \Pi = 0 \\ & \mathrm{tr}(\Pi) \leq \pi_{\max} \\ & f_o \leq f_{o,\max} \\ & f_{c,\min} \leq f_c \leq f_{c,\max} \\ & 0 < \alpha < 1. \end{aligned} \tag{24}$$

In (24), μ_o and μ_c are the costs associated with the observation and communication rates, respectively. The uncertainty criterion is established with the first two constrains. The stationary estimation covariance Π is characterized by the CARE (18), and the accuracy requirement is presented by the second constrain, where the trace of the stationary estimation covariance should be bounded by $\pi_{\max}$. The maximum observation rate $f_{o,\max}$ and the maximum communication rate $f_{c,\max}$ represent the physical limitation of the underlying operation. The last constraint is the requirement from CI fusion on α.

The inequality $f_{c,\min} \leq f_c$ is imposed to account for the imperfection of the continuous-time approximation. According to the investigation in Section V, increasing the communication rate f_c may lead to the increase of Π, and thus the optimal f_c may be relatively small; this results in large amplitude steps in the covariance of the corresponding discrete-time setting. With this large sawtooth behavior in the discrete-time case, our continuous-time approximations, including the reordering of operations especially, become suspect.

B. Trace Minimization Problem

We can also establish the optimal scheduling problem by minimizing the localization uncertainty given an overall cost budget $\mu_{\max}$, as shown in the following:

$$\begin{aligned} \underset{f_o, f_c, \alpha}{\text{minimize}} \quad & \mathrm{tr}(\Pi) \\ \text{subject to} \quad & 2 f_c \left(\frac{1}{\sqrt{\alpha}} - 1 \right) \Pi + Q_c \\ & \quad - \Pi \check{H}^\mathsf{T} \check{R}_c^{-1}(f_o, f_c, \alpha) \check{H} \Pi = 0 \\ & \mu_o f_o + \mu_c f_c \leq \mu_{\max} \\ & f_o \leq f_{o,\max} \\ & f_{c,\min} \leq f_c \leq f_{c,\max} \\ & 0 < \alpha < 1. \end{aligned} \tag{25}$$

Since the feasible set of parameters is bounded, the global optimum of both optimization problems can be obtained by exhaustive search. More sophisticated solving techniques may be explored using a further understanding of the properties of both optimization problems.

VII. An Example of Optimal Scheduling

We demonstrate the effectiveness of reducing the operation cost or achieving better estimation performance by solving the

TABLE I
PARAMETERS OF REAL SCENARIO FOR SCHEDULING EXAMPLES

observation cost	μ_o	20 mW
maximum observation rate	$f_{o,\max}$	3 Hz
maximum observation distance	$d_{\max}$	4.8 m
the variance of range sensing	σ_d^2	0.0215 m^2
the variance of bearing sensing	σ_ϕ^2	0.01 rad^2
communication cost	μ_c	100 mW
maximum communication rate	$f_{c,\max}$	10 Hz
minimum communication rate	$f_{c,\min}$	0.2 Hz
propagation rate	f_p	10 Hz

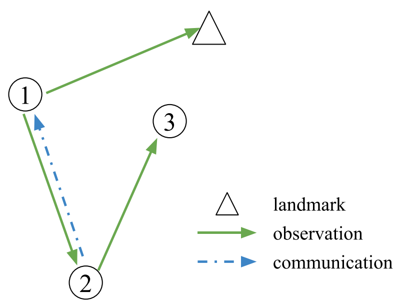

Fig. 1. The topology of the simulated multirobot system.

optimization problems on a simulated system with parameters listed in Table I. We consider three robots, indexed by 1, 2, and 3. Robot 1 can observe the landmark and robot 2; robot 2 can observe robot 3 and sends its information to robot 1, as the complete topology in Fig. 1. Based on the analysis in [1], the estimation covariance of robot 1 is bounded, even though that of robot 2 is not.

A typical example scenario is presented as a baseline with $f_c = 1$ Hz, $f_o = 2$ Hz. The total cost is $\mu = 140$ mW, and the trace of stationary covariance is $\text{tr}(\Pi) = 2.0918$ m^2 with an optimized $\alpha = 0.90$. With this baseline example, we can consider both the cost minimization problem (24) with the same performance criterion, and the trace minimization problem (25) with the same amount of cost budget $\mu_{\max} = 140$ mW, respectively.

A. Experimental Results

We plot the trace of the stationary covariance with optimized α in Fig. 2, and we arrive the optimal solution for each optimization problem. The optimized parameters and the optimization result of two scenarios are listed in Table II for comparison. After solving the cost minimization problem, the cost reduction is around 64% for the same error performance, achieved by reducing the communication rate f_c and CI parameter α jointly. In the trace minimization problem, we can instead reduce 15% of the stationary covariance trace within the same original resource budget, in fact simultaneously decreasing cost by 43%.

With the parameters obtained in continuous-time approximation, we can apply them in the original discrete-time setting. The simulation with baseline scenario, cost minimization and trace minimization is presented in Fig. 3, where the operation parameters are obtained from Table II. The estimation error is defined as $||\hat{s}^1 - s||_2$. We plot an average over 10 sampled curves in Fig. 3 and present the result in Table III, where the estimation error and the covariance trace are averaged over the operation period. From Fig. 3, as $f_c = 0.2$ in both optimized case, we can observe that the error curves in these two cases follow a sawtooth profile in accordance with the communication period. This motivates the constraint $f_{c,\min}$ in the optimization problems (24) and (25). Also, the discrete-time realization reflects

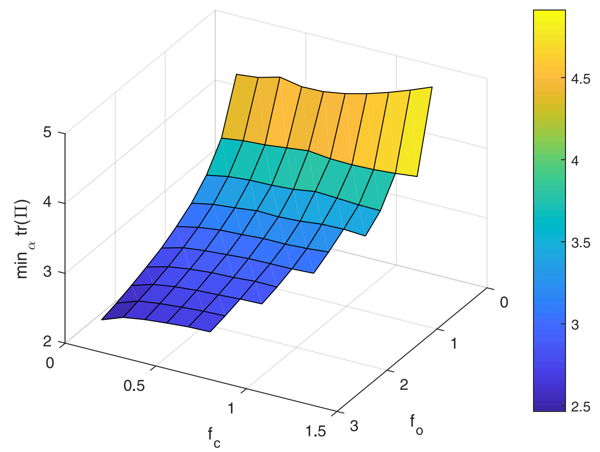

Fig. 2. The plot of $\min_\alpha \text{tr}(\Pi)$ in the region with feasible operation rates.

TABLE II
RESULTS FOR SCHEDULING EXAMPLES IN CONTINUOUS-TIME APPROXIMATION

scenario	f_o [Hz]	f_c [Hz]	α	μ [mW]	tr(Π) [m^2]
baseline	2	1	0.90	140	2.9018
cost min (24)	1.52	0.2	0.63	50.4	2.9003
trace min (25)	3	0.2	0.47	80	2.4617

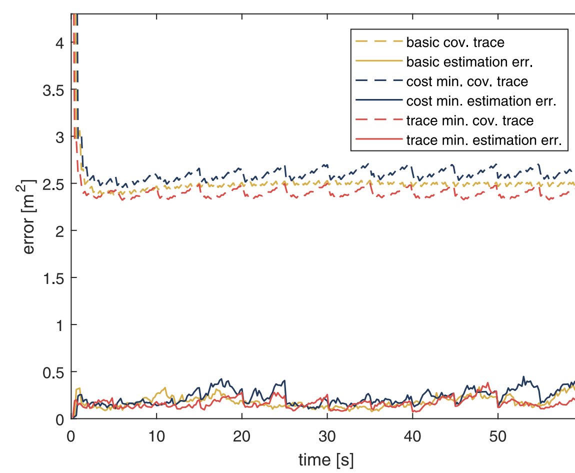

Fig. 3. The discrete-time simulation with parameters from optimal scheduling result.

TABLE III
RESULTS FOR SCHEDULING EXAMPLES IN DISCRETE-TIME SIMULATION

scenario	estimation error [m^2]	cov. trace [m^2]	power [mW]
baseline	0.1894	2.5051	140
cost min (24)	0.2365	2.6251	50.4
trace min (25)	0.1615	2.4070	80

the design goal in the continuous-time counterpart. The two examples show the desired improvement in terms of the cost and of the stationary trace by considering the optimal scheduling problems.

B. Discussion

In the trace optimization problem, the communication rate is small comparing to the baseline case. The saving of communication operation with the benefit of trace improvement may seem confusing at first. However, since the communication update applies covariance intersection, the multiplication of α does increase the covariance. This phenomenon can also be explained by the discussion in Section V. With the analysis of the Fréchet derivative, from $f_c = 0.2$ to $f_c = 1$, the contribution of communication incoming information is compensated by the uncertainty introduced by CI algorithm. This example not only demonstrates the improvement by simply engineering the parameters, it also shows the necessity of optimizing operation parameters in such scheme.

While this example presents a minimum topology to clearly demonstrate our algorithm, the extension to general topologies is direct. Equation (12) can account for arbitrary observation topology without difficulty via (6) and (10). For the communication topology, two types of generalization are considered for completeness. If the incoming information contains other communicated data, as discussed in Section IV, (12) can simply be extended to include the additional terms. If there are multiple incoming communication paths, from several robots transmitting their data to the receiving node, CI will use a set of convex coefficients instead of a single coefficient α for such data fusion. In this case, the number of optimization variables will increase, but the formulation remains unaltered.

One should note that the operation periods, for example T_o and T_c, may not have a common divisor in general, which makes discrete-time analysis intractable. Nevertheless, as demonstrated by this example, we show that it is straightforward to design and to analyze these operation periods in a corresponding continuous-time limiting approximation. The tightness of the upper bound mainly depends on the knowledge of odometry and observation error. One can further improve those bounds with more available information, and optimize the localization algorithm following the procedure in this letter without changing any step.

VIII. Conclusion

While implementing localization in multirobot systems, this paper shows that one can achieve comparable accuracy performance at lower cost. This is crucial for the sustainability of systems with constrained resources, such as battery powered robots, using expensive operations, such as power-hungry wireless communications. By running the numerical optimization as presented with parameters as determined from physical hardware, a systems engineer can optimally schedule individual sensing and communications operations based on operational requirements to maximize effectiveness.

In addition, the analysis shown in this letter is not limited to localization algorithms. For arbitrary distributed estimation problems involving multiple operations with distinct costs and uncertainties, we can formulate an optimization problem using the presented continuous-time approximation procedure together with the analysis from the Fréchet derivative. By applying the procedure in this letter, the operation parameters can then be optimized for performance improvement.

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