Multirobot Cooperative Localization Algorithm with Explicit Communication and its Topology Analysis

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Abstract This paper proposes a new cooperative localization algorithm that separates communication and observation into independent mechanisms. While existing algorithms acknowledge observations between robots are crucial in cooperative localization schemes, communication is considered only an auxiliary role in observation update but not explicitly stated. However, such algorithms require the communication to be available whenever needed, and it is difficult to consider the effect of communication imperfection, which is unavoidable in real systems. We propose the Global State–Covariance Intersection (GS-CI) multirobot cooperative localization algorithm that can independently update localization estimates through both observation and communication steps. We also provide a theoretical upper bound of the resulting estimation uncertainty based on observation and communication topologies. Simulations using generated data validates the theoretical analysis, and shows the comparable performance to the centralized equivalent approach with less communication together with real-world data.

1 Introduction

Localization is one of the most fundamental tasks for mobile robots [18]. In multirobot systems, robots can observe and communicate with one another to achieve localization cooperatively, even in uncharted places or GPS-denied environments. This scheme is called cooperative localization (CL), and is considered a promising

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approach for better autonomy. Among all approaches for CL, we focus on those based on the extended Kalman filter (EKF) due to its computational efficiency.

The success of CL algorithms primarily lies in tracking the correlations between robots. If the correlations between robots are underestimated, that is, two estimates are considered more independent than they really are, the uncertainty in the resulting localization estimation may be too optimistic. This is exactly the double-counting problem, or *over-convergence problem*, which impairs the localization reliability. Consequently, extensive communication between the distributed robots is required to track those correlations across all localization estimates.

Besides the communication overheads in these algorithms, there is a more fundamental concern: the communication is often implicitly assumed to be perfect after relative observations. In fact, observation and communication are performed by different hardware modules, and communication failures are unavoidable in real systems. That is, the availabilities of observation and of communication are not identical, and under this assumption the observation step is vulnerable to communication failure.

In this paper, we propose the Global State–Covariance Intersection (GS-CI) multirobot cooperative localization algorithm that separates communication and observation as two independent information contributors. As a consequence, the proposed algorithm needs lower communication cost, and is robust to communication failure. To avoid tracking the exact interdependency between distributive estimations, we apply covariance intersection (CI) [1, 5, 16] during the communication update to get a conservative but consistent result, which guarantees that the computed covariance is always not smaller than the true covariance in a positive-definite sense.

While the framework of the proposed algorithm is similar to the ones discussed in vehicular technology [9, 13], we furthermore provide the theoretical treatment for the proposed algorithm. It has been shown in [15] that in a centralized-equivalent localization algorithm, the covariance will be bounded if at least one landmark are observed. The verification should also be given for the proposed algorithm, concerning the disagreement of observation and communication topologies, and the conservative fusion result by CI as well.

The contributions of this paper include:

- The CL algorithm that separates out communications explicitly and also preserves estimation consistency to avoid over-convergence
- An analysis of the boundedness properties of the algorithm with respect to observation and communication topologies
- Comparisons of our algorithm vs. other state-of-the-art algorithms based on simulated and measured data demonstrating comparable estimation error with less communication cost

This paper is organized as follow: Related work is presented in the following section. We setup the system model in Section 3. The main algorithm is proposed in Section 4, while the associative topology analysis is given in Section 5. Simulations are delivered in Section 6, followed by the conclusions of this paper.

2 Related Work

The benchmark of EKF-CL is established in [17], together with the theoretical analysis in [15]. The formulation in [17] begins with a strict requirement on the availability of the local information in each robots, which can only be satisfied in a centralized system. A distributive version is then designed in the same paper, where each robot holds its own estimation and correlation parameters regarding rest of the robots. However, to keep those correlation parameters in each robot updated, the communication complexity of this algorithm after one relative observation is $O(N^2)$, where N is the number of the robots. The communication complexity is lowered to $O(N)$ in [11] by introducing the interim master, who is in charge of calculating and broadcasting the required updating information for the rest of the robots. Following the same modeling, a partially-decentralized scheme is discussed in [10], mainly to combat information dropouts in communication failure. However, a central process unit in charge of information exchange is indispensable in this scheme.

In [3, 14], the communication cost is only $O(1)$ after the relative observation. In both papers, only the estimation information in the robots involved in relative observation is utilized, which means that the exact correlation between the fused estimates is not available but can only be approximate. In other words, the communication cost is lowered with the cost of estimation accuracy. In [3], CI is applied to get a conservative but consistent estimation result. While our algorithm applies CI as well, it outperforms the one proposed in [3] in estimation accuracy since our algorithm can utilize the correlations of robots not involved in the relative observation. In [14], the exact correlation between the fused estimates is approximated without the guarantee of estimation consistency. As mentioned in [14], the approximated covariance in their algorithm may be smaller than the real covariance in positive-definite sense, which may lead to over-convergence problem. For all the aforementioned algorithms [17, 3, 14], the communication takes place directly after relative observation. On the contrary, we separate communication and observation for the robustness against communication failure.

Similar to the algorithm we propose, the CL algorithm for vehicle networks stores the estimation for the whole system in each agent, called *state exchange scheme* [8, 9, 13]. The main challenge of the state exchange scheme is to avoid over convergence problem without the knowledge of correlation between state estimates. In [8, 9], only independent estimates are fused. However, to keep the required independence, historical information is discarded if there are new-coming independent information. The split covariance intersection (SCI) is applied in [13] in information fusion. One of the main drawbacks of this approach is that SCI is not applicable in relative observation due to the dependency of the information, as mentioned in [3]. Instead of applying SCI, we will consider CI for generalization without imposing further requirement. Furthermore, we are the only one that provides theoretical argument to guarantee the consistency and boundedness of the uncertainty, to the best of our knowledge.

As for the estimation algorithm, our framework is similar to the diffusion Kalman filter with CI [6]. In the diffusion step of the original diffusion Kalman filter [4], the combination of estimations is based on heuristic convex combination without updating the corresponding covariance matrix. In [6], the diffusion step is replaced by CI, which guarantees the consistency of the resulting estimation. The major difference between our algorithm and diffusion-based KF is that we do not implement incremental update, where the raw measurement data is exchanged and incorporated. However, whether to exchange state estimates or measurement results or both should be further investigated concerning the accuracy and efficiency jointly.

3 System Model

We consider a 2D multirobot system indexed by $\Omega = \{1, \ldots, N\}$, together with several landmarks whose locations are known by the robots in advance. Landmarks are denoted as Δ, and $\Omega^* = \Omega \cup \{\Delta\}$. The position of robot i at time t is regarded as the state, denoted as $s_{i,t} = [x_{i,t}, y_{i,t}]^\mathsf{T}$, where T denotes matrix transpose. The orientation of robot i at time t is denoted by $\theta_{i,t}$, and we do not incorporate $\theta_{i,t}$ in the estimation state due to the linearization issue [2]. The state of the whole system is denoted by $s_t = [s_{1,t}^\mathsf{T}, \ldots, s_{N,t}^\mathsf{T}]^\mathsf{T}$. In EKF, each robot i keeps an estimate of s_t, denoted by $\hat{s}_t^i$, together with its covariance $\Sigma_{s^i,t}$.

3.1 Motion Model

The motion model describes the spatial displacement of robots due to odometry inputs. While the framework is not limited to any specific models, we mainly consider the velocity input $v_{i,t}$ in this paper. Let δt be the time interval between two consecutive observation points, the state of robot i at the next time is given by

$$s_{i,t+1} = f_i(s_{i,t}, v_{i,t}) = \begin{bmatrix} x_{i,t} + v_{i,t}\delta t \cos(\theta_{i,t}) \\ y_{i,t} + v_{i,t}\delta t \sin(\theta_{i,t}) \end{bmatrix}. \tag{1}$$

3.2 Observation Model

If robot i observes an object j, either a robot or a landmark, the relative position obtained by robot i is

$$o_{ij} = C^\mathsf{T}(\theta_{i,t}) \left(\begin{bmatrix} x_{j,t} \\ y_{j,t} \end{bmatrix} - \begin{bmatrix} x_{i,t} \\ y_{i,t} \end{bmatrix} \right) = H_{o_{ij}} s_t, \tag{2}$$

where $C(\theta)$ is the rotation matrix with argument θ.

Most of the time, the relative positions can not be obtained directly, but they are general enough to incorporate different kinds of sensing result. The observation is often accomplished by distance and bearing sensors. Consequently, the relative position can also be expressed as

$$o_{ij} = d_{ij} \begin{bmatrix} \cos(\phi_{ij}) \\ \sin(\phi_{ij}) \end{bmatrix}, \tag{3}$$

based on the relative distance d_{ij} and relative bearing ϕ_{ij}.

4 Cooperative Localization Algorithm with Explicit Communication

As mentioned, the CL algorithm should consist of three parts based on the hardware module. In this section, we present an algorithm according to these three actions: motion propagation, observation and communication. Motion propagation and observation steps are standard EKF updates, but we have to handle the unavailable odometry inputs of other robots in motion propagation. In communication step, it is nothing more than a direct application of CI, which will be introduced in the following.

4.1 Motion Propagation

We consider robot i for example. While the velocity inputs of each robot are uncorrelated, the covariance update of motion propagation is

$$\Sigma_{s^i,t+1} = \Sigma_{s^i,t} + \Sigma_{q_i} = \Sigma_{s^i,t} + \mathsf{Diag}(\Sigma_{u_1}, \ldots, \Sigma_{u_N}). \tag{4}$$

The determination of Σ_{u_j} depends on the velocity inputs v_j for all j. For robot i, the velocity input v_i is available, and it is disturbed by a noise $\mathbf{n}_v$ modeled as zero-mean Gaussian random variable with variance $\sigma^2_{\mathbf{n}_v}$. By linearizing (1), the error propagation equation of robot i itself is

$$\tilde{s}_{i,t+1} \approx \begin{bmatrix} \tilde{x}_{i,t} \\ \tilde{y}_{i,t} \end{bmatrix} + \delta t \begin{bmatrix} \cos(\theta_{i,t}) & -v_i \sin(\theta_{i,t}) \\ \sin(\theta_{i,t}) & v_i \cos(\theta_{i,t}) \end{bmatrix} \begin{bmatrix} \mathbf{n}_v \\ \tilde{\theta}_{i,t} \end{bmatrix}. \tag{5}$$

The errors in the orientation estimates $\tilde{\theta}_{i,t} = \theta_{i,t} - \hat{\theta}_{i,t}$ are modeled by a zero-mean Gaussian random variable, whose variance $\sigma^2_{\tilde{\theta}_{i,t}} = \mathsf{E}[\tilde{\theta}^2_{i,t}]$ is bounded by $\sigma^2_{\tilde{\theta}_i}$. The increment of covariance matrix Σ_{u_i} can then be obtained as

$$\Sigma_{u_i} = (\delta t)^2 C(\theta_{i,t}) \begin{bmatrix} \sigma^2_{\mathbf{n}_v} & 0 \\ 0 & v_i^2 \sigma^2_{\tilde{\theta}_i} \end{bmatrix} C^{\mathsf{T}}(\theta_{i,t}). \tag{6}$$

The velocity inputs $v_j, j \neq i$, are not available for robot i. Without the exact value, we than model the input itself as a Gaussian random variable $\mathbf{v}_j$, whose variance $\sigma_{\mathbf{v}}^2$ can be determined by the maximum input value $v_{\max}$ and $\sigma_{\mathbf{n}_v}^2 < \sigma_{\mathbf{v}}^2$. While the exact velocity value is unknown, we can still get upper bound covariance increment by

$$\Sigma_{u_j} = (\delta t)^2 \max(\sigma_{\mathbf{v}}^2, v_{\max}^2 \sigma_{\tilde{\theta}_i}^2) I_2, \quad j \neq i, \tag{7}$$

where I_n is the $n \times n$ identity matrix.

4.2 Observation

The observation step updates the estimates based on the exteroceptive measurements o_{ij}, which is standard in EKF procedure. The subscript i is omitted in the following derivation since the observation update occurs only in the robot itself. Based on (2), the innovation $\tilde{o}_{ij} = o_{ij} - \hat{o}_{ij}$ can be approximated as

$$\tilde{o}_{ij} \approx H_{o_{ij}} \tilde{s} + C^{\mathsf{T}}(\hat{\theta}_i) J H_{ij} \hat{s}_{t^-} \tilde{\theta}_i + \mathbf{n}_{o_{ij}}, \tag{8}$$

to distinguish the estimation error $\tilde{s}$, the orientation estimation error $\tilde{\theta}_i$, and the measurement noise $\mathbf{n}_{o_{ij}}$, where $H_{o_{ij}} = C^{\mathsf{T}}(\theta_i) H_{ij}$ and $J = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$. If the object j is a landmark,

$$H_{ij} = \begin{bmatrix} 0_{2\times 2} & \cdots & \underbrace{-I_2}_{i} & \cdots & 0_{2\times 2} \end{bmatrix}_{2\times 2N};$$

while the object is robot j, then

$$H_{ij} = \begin{bmatrix} 0_{2\times 2} & \cdots & \underbrace{-I_2}_{i} & \cdots & \underbrace{I_2}_{j} & \cdots & 0_{2\times 2} \end{bmatrix}_{2\times 2N}.$$

Furthermore, if the observed relative position is obtained by distance and bearing sensors as in (3), the errors in observed relative position $\mathbf{n}_{o_{ij}}$ can be expressed as

$$\mathbf{n}_{o_{ij}} = \begin{bmatrix} \cos\phi_{ij} & -d_{ij}\sin\phi_{ij} \\ \sin\phi_{ij} & d_{ij}\cos\phi_{ij} \end{bmatrix} \begin{bmatrix} \tilde{d}_i \\ \tilde{\phi}_i \end{bmatrix},$$

with the corresponding covariance $R_{o_{ij}}$. Then the covariance of innovation $\tilde{o}_{ij}$ is $\Sigma_{o_{ij}} = \mathsf{E}[\tilde{o}_{ij}\tilde{o}_{ij}^{\mathsf{T}}] = H_{o_{ij}} \Sigma_{s,t^-} H_{o_{ij}}^{\mathsf{T}} + R_{\theta_{ij}} + R_{o_{ij}}$, where $R_{\theta_{ij}} = G_{ij} \sigma_{\theta_i}^2 G_{ij}^{\mathsf{T}}$, $G_{ij} = C^{\mathsf{T}}(\hat{\theta}_i) J H_{ij} \hat{s}_{t^-}$.

Robots can observe more than one object at the same time, and the observation results are then correlated. Therefore, we should process the results in the same observation step together. We first define the observation topology $\mathscr{O}_{i,t} = \{\Omega^*, E_{\mathscr{O}_{i,t}}\}$

as a directed graph with edges $(i,j) \in E_{\mathscr{O}_{i,t}}$ where j is the object observed by i at time t. For simplicity, $O^*_{i,t} = \{j \in \Omega^* | (i,j) \in E_{\mathscr{O}_{i,t}}\}$ is the index set of objects observed by robot i, and $O_{i,t} = O^*_{i,t} \cap \Omega$. The overall observation result can be presented by the stacked vector

$$o_i = [o_{ij}]_{j \in O^*_{i,t}} = [o^{\mathsf{T}}_{ij_1}, \ldots, o^{\mathsf{T}}_{ij_k}]^{\mathsf{T}}, \quad j_1, \ldots, j_k \in O^*_{i,t}.$$

By separating state error and the rest terms, the covariance of the innovation $\tilde{o}_i$ is $\Sigma_{o_i} = \mathbf{H}_{o_i} \Sigma_{s,t^-} \mathbf{H}^{\mathsf{T}}_{o_i} + \Sigma_{r_i}$, where $\mathbf{H}_{o_i} = [H_{o_{ij}}]_{j \in O^*_{i,t}}$. Explicitly, $\Sigma_{r_i} = \mathbf{G}_i \sigma^2_{\theta_i} \mathbf{G}^{\mathsf{T}}_i + \mathsf{Diag}(R_{o_{ij}})$, where $\mathbf{G}_i = [G_{ij}]_{j \in O^*_{i,t}}$.

4.3 Communication

When robot j sends its estimation information, in particular $\hat{s}^j$ and Σ_{s^j}, to robot i, robot i can use this information to update its own estimation. However, the correlation between $\hat{s}^i$ and $\hat{s}^j$ is not recorded at all. Without knowing the correlation among estimates, the fusion of estimates becomes challenging. In particular, if one underestimates the correlation between two fused estimates, or the two estimates are considered more independent than they really are, one encounter the over-convergence problem. However, tracking the correlations imposes excessive communication overhead as in aforementioned algorithms [17, 3, 14]. Here, we apply CI to fuse two estimates without knowing the correlation between them, and to guarantee the consistency of estimates.

Definition 1 (Consistent fusion [1]). An estimate z is a Gaussian random vector with mean $\mathsf{E}[z] = \bar{z}$ and covariance Σ_z. The pair $(\hat{z}, \Sigma_{\hat{z}})$ is called *consistent* if $\mathsf{E}[\hat{z}] = \bar{z}$ and $\Sigma_{\hat{z}} \geq \Sigma_z$.

A consistent estimate can be regarded as another estimate which is mean-preserving but conservative in terms of covariance matrix.

We can similarly define the the communication topology $\mathscr{C}_{i,t} = \{\Omega^*, E_{\mathscr{C}_{i,t}}\}$ as a directed graph with edges as $(j,i) \in E_{\mathscr{C}_{i,t}}$ where j sends its information to i at time t. Also, $C_{i,t} = \{j \in \Omega | (j,i) \in E_{\mathscr{C}_{i,t}}\}$ is the index set of the information sources. By directly applying CI, we have the update equation for robot i when receiving information from $j \in C_{i,t}$, the index set of the robots sending information to robot i, as in (13) and (14).

The coefficients $\{c_0, c_j, j \in C_{i,t}\}$ are all nonnegative and sum to 1. The determination of the coefficient is usually based on the minimization of $\mathsf{det}(\Sigma_{s^i,t^+})$ or that of $\mathsf{tr}(\Sigma_{s^i,t^+})$. In implementation, the communication update normally involves two estimates only, since the inverse of convex combination parameters may lead to undesirably conservative result once the number of parameters is large.

Algorithm 1 Global State Covariance Intersection (GS-CI) Multirobot Cooperative Localization Algorithm

Initialization

Set $\hat{s}^i_0$ and $\Sigma_{s^i,0}$ for robot i.

Motion Propagation

input: odometry input $v_{i,t}$

$$\hat{s}^i_{t+1} = [f_1^\mathsf{T}(\hat{s}^i_{1,t}, \mathsf{E}[\mathbf{v}_{1,t}]), \ldots, f_i^\mathsf{T}(\hat{s}^i_{i,t}, v_{i,t}), \ldots, f_N^\mathsf{T}(\hat{s}^i_{N,t}, \mathsf{E}[\mathbf{v}_{N,t}])]^\mathsf{T}, \tag{9}$$

$$\Sigma_{s^i,t+1} = \Sigma_{s^i,t} + \mathsf{Diag}(\Sigma_{u_1}, \ldots, \Sigma_{u_N}). \tag{10}$$

Observation

input: observation result o_i

$$\hat{s}^i_{t^+} = \hat{s}^i_{t^-} + \Sigma_{s^i,t^-}\mathbf{H}_{o_i}^\mathsf{T}\Sigma_{o_i}^{-1}(o_i - \mathbf{H}_{o_i}\hat{s}^i_{t^-}), \tag{11}$$

$$\Sigma_{s^i,t^+} = \Sigma_{s^i,t^-} - \Sigma_{s^i,t^-}\mathbf{H}_{o_i}^\mathsf{T}\Sigma_{o_i}^{-1}\mathbf{H}_{o_i}\Sigma_{s^i,t^-}. \tag{12}$$

Communication

input: $\hat{s}^j_{t^-}$, Σ_{s^j,t^-} from robot $j \in C_{i,t}$

$$\hat{s}^i_{t^+} = \Sigma_{s^i,t^+}\left[c_0\Sigma^{-1}_{s^i,t^-}\hat{s}^i_{t^-} + \sum_{j\in C_{i,t}} c_j\Sigma^{-1}_{s^j,t^-}\hat{s}^j_{t^-}\right], \tag{13}$$

$$\Sigma^{-1}_{s^i,t^+} = c_0\Sigma^{-1}_{s^i,t^-} + \sum_{j\in C_{i,t}} c_j\Sigma^{-1}_{s^j,t^-}. \tag{14}$$

4.4 Summary

The overall proposed algorithm is summarized in Algorithm 1, Global State Covariance Intersection (GS-CI) Multirobot Cooperative Localization Algorithm. The term global state indicates that each robot keeps an estimate on the whole system, in order to distinguish other algorithms in which each robot only keeps an estimate on its own state.

5 Topological Analysis of Global State Covariance Intersection Multirobot Cooperative Localization Algorithm

In EKF setup, the uncertainty of the estimate $\hat{s}^i$ is represented by the state covariance Σ_{s^i}. While the observation and communication topologies are no longer required to be identical in the proposed algorithm, together with conservative fusion scheme in CI, the uncertainty of the proposed algorithm should be guaranteed non-divergent as a prerequisite for further application, as in [15]. Therefore, we determine the boundedness criteria for Σ_{s^i} of the proposed algorithm in this section, and it mainly depends on the observation and communication topology $(\mathscr{O}_{i,t}, \mathscr{C}_{i,t})$. To proceed, we use the notation convention that for $P > 0$ and an index set I, we define $[P]_I$ as the submatrix of the index set I.

We consider a time-slotted scenario for theoretical analysis. In each time slot, every robot executes motion propagation, observation and communication in Algorithm 1 consecutively. We assume that the observation topology $\mathscr{O}_i$ and communication configuration $\mathscr{C}_i$ are time-invariant for each robot. The parameters in convex combination are also constant w.r.t. time. We then obtain the covariance update in each time slot as

$$\Sigma_{s^i,t+1} = \left[c_0\Sigma_{s^i,t}^{-1} + c_0\mathbf{H}_{o_i}^\top\Sigma_{r_i}^{-1}\mathbf{H}_{o_i} + \sum_{j\in C_i} c_j\Sigma_{s^j,t}^{-1}\right]^{-1} + \Sigma_{q_i}. \tag{15}$$

With the definition $\mathbf{H}_i = [H_{ij}]_{j\in O_i^*}$, it follows that $\mathbf{H}_{o_i} = \Xi^\top(\hat{\theta}_i)\mathbf{H}_i$, $\Xi(\hat{\theta}_i) = I_N \otimes C(\hat{\theta}_i)$, where $\otimes$ stands for Kronecker multiplication. The update equation (16) can then be rewritten as

$$\Sigma_{s^i,t+1} = \left[c_0\Sigma_{s^i,t}^{-1} + c_0\mathbf{H}_i^\top\Xi(\hat{\theta}_i)\Sigma_{r_i}^{-1}\Xi^\top(\hat{\theta}_i)\mathbf{H}_i + \sum_{j\in C_i} c_j\Sigma_{s^j,t}^{-1}\right]^{-1} + \Sigma_{q_i}. \tag{16}$$

We can find the covariance bound of the error in observation and in motion propagation, Σ_r and Σ_q respectively, such that $\Sigma_r \geq \Xi^\top\Sigma_{r_i}\Xi$ and $\Sigma_q \geq \Sigma_{q_i}$ by some fairly common physical constraints. The detail can be found in [15]. By substituting the corresponding terms in (16), we can find another sequence $\Pi_{s^i,t}$ with recursion

$$\Pi_{s^i,t+1} = \left[c_0\Pi_{s^i,t}^{-1} + c_0\mathbf{H}_i^\top\Sigma_r^{-1}\mathbf{H}_i + \sum_{j\in C_i} c_j\Pi_{s^j,t}^{-1}\right]^{-1} + \Sigma_q. \tag{17}$$

such that $\Pi_{s^i,t} \geq \Sigma_{s^i,t}$ for all t with the same initial condition $\Pi_{s^i,0} = \Sigma_{s^i,0}$. Furthermore, in (17) every term besides $\Pi_{s^i,t}$ is constant and independent of the state of the system.

Proposition 1. *If $\Delta \in O_i^*$, then $[\Pi_{s^i,t}]_{O_i}$ is bounded with positive definite initial condition.*

Proof. We first rearrange the order in $\Pi_{s^i,t+1}$ to pick i as the first, followed by other terms in O_i, while the rest are all indices not in O_i. Thus, $[\Pi_{s^i,t+1}]_{O_i}$ is now the upper-left diagonal submatrix of $\Pi_{s^i,t+1}$.

In fact, the elements in $\mathbf{H}_i^\top\Sigma_r^{-1}\mathbf{H}_i$ are zero outside O_i. Explicitly,

$$[\mathbf{H}_i^\top\Sigma_r^{-1}\mathbf{H}_i]_{O_i} = \begin{bmatrix} n & -\mathbf{1}_{n-1}^\top \\ -\mathbf{1}_{n-1} & I_{n-1}\end{bmatrix} \otimes \Sigma_r^{-1},$$

where $n = |O_i|$. With the special structure, the matrix is nonsingular, with the inversion

$$R = [\mathbf{H}_i^\top\Sigma_r^{-1}\mathbf{H}_i]_{O_i}{}^{-1} = \left(\mathbf{1}_n\mathbf{1}_n^\top + \begin{bmatrix} 0 & 0 \\ 0 & I_{n-1}\end{bmatrix}\right) \otimes \Sigma_r.$$

By dropping positive definite $\sum_{j\in C_i} c_j \Pi_{s^j,t}^{-1}$ in the inversion, we have

$$\Pi_{t+1} \leq \left[c_0 \Pi_t^{-1} + c_0 \mathbf{H}_i^\mathsf{T} \Sigma_r^{-1} \mathbf{H}_i\right]^{-1} + \Sigma_q. \tag{18}$$

Applying Lemma 1 in Appendix, the upper-left diagonal term in the inversion satisfies

$$\begin{aligned} c_0[\Pi_t^{-1} + \mathbf{H}_i^\mathsf{T} \Sigma_r^{-1} \mathbf{H}_i]_{O_i} &= c_0[\Pi_t^{-1}]_{O_i} + c_0[\mathbf{H}_i^\mathsf{T} \Sigma_r^{-1} \mathbf{H}_i]_{O_i} \\ &\geq c_0 [\Pi_t]_{O_i}{}^{-1} + c_0 R^{-1}. \end{aligned}$$

We can choose $B = [c_0 \Pi_t^{-1}]_{(O_i)^\mathsf{C}}$, then by Lemma 2

$$c_0 \Pi_t^{-1} + c_0 \mathbf{H}_i^\mathsf{T} \Sigma_r^{-1} \mathbf{H}_i \geq \frac{1}{2} \operatorname{Diag}\left(c_0 [\Pi_t]_{O_i}{}^{-1} + c_0 R^{-1}, B\right).$$

The original recursion can then be bounded by

$$\Pi_{t+1} \leq 2 \operatorname{Diag}\left(\left[c_0 [\Pi_t]_{O_i}{}^{-1} + c_0 R^{-1}\right]^{-1}, B^{-1}\right) + \Sigma_q,$$

which implies the solution P_t of the recursion

$$\begin{aligned} P_{t+1} &= 2\left[c_0 P_t^{-1} + c_0 R^{-1}\right]^{-1} + [\Sigma_q]_{O_i} \\ &= \frac{2}{c_0}\left[P_t - P_t (R + P_t)^{-1} P_t\right] + [\Sigma_q]_{O_i} \end{aligned}$$

is always larger than $[\Pi_t]_{O_i}$, and P_t converges by [7, Ch. 14.5].

Proposition 1 indicates that whenever one robot observes a landmark, the error in its estimates of those observed robot together with that of the robot itself will be bounded.

Proposition 2. *Suppose that $\Delta \in O_i^* \cup O_j^*$ and $O_i^* \cap O_j^* \neq \emptyset$. If $j \in C_i$, then $[\Pi_{s^i,t}]_{O_i \cup O_j}$ will be bounded given positive definite initial condition.*

Proof. By dropping the communication terms other than robot j in (17), we have

$$\Pi_{s^i,t+1} \leq \left[c_0 \Pi_{s^i,t}^{-1} + c_0 \mathbf{H}_i^\mathsf{T} \Sigma_q^{-1} \mathbf{H}_i + c_j \Pi_{s^j,t}^{-1}\right]^{-1} + \Sigma_u. \tag{19}$$

If $\Delta \in O_i^*$ and $\Delta \in O_j^*$, we can easily obtain the conclusion from Proposition 1 and Lemma 2. Therefore, we only consider that Δ lies in one set.

Given $\Delta \in O_j^*$ and $\Delta \notin O_i^*$, we now consider a submatrix of $\Pi_{s^i,t}$ with index in $O_i \cup O_j$. From Proposition 1, we know that $[\Pi_{s^j,t}]_{O_j}$ is bounded, then so is $[\Pi_{s^i,t}]_{O_j}$. To find the bound for $[\Pi_{s^i,t}]_{O_i}$, we first note that with $n = |O_i|$,

$$[\mathbf{H}_i^\mathsf{T} \Sigma_r^{-1} \mathbf{H}_i]_{O_i} = \begin{bmatrix} n & -\mathbf{1}_n^\mathsf{T} \\ -\mathbf{1}_n & I_n \end{bmatrix} \otimes \Sigma_r^{-1},$$

which is singular. We then find one overlapping index $k \in O_i \cap O_j$, and $[\Pi_{s^i,t}]_{\{k\}}$ is bounded since $k \in O_j$. By deleting the row and the column associated with k, $[\mathbf{H}_i^\mathsf{T} \Sigma_r^{-1} \mathbf{H}_i]_{O_i \setminus \{k\}}$ is now nonsingular, which leads to the boundedness of $[\Pi_{s^i,t}]_{O_i \setminus \{k\}}$. By Lemma 2, the conclusion follows.

The case $\Delta \in O_i^*$ and $\Delta \notin O_j^*$ can be proved in the similar way by replacing the communication term $\Pi_{s^j,t}$ in (19) with the covariance update equation of j.

Proposition 2 can be easily extended to $[\Pi_{s^i,t}]_{O_i \bigcup_{j \in \mathscr{C}_i} O_j}$, if $\Delta \in O_i^* \bigcup_{j \in \mathscr{C}_i} O_j^*$ and $O_i^* \cap O_j^* \neq \emptyset$ for every pair of robots.

We can restate Propositions 1 and 2 in graph perspective to understand the communication effect on the estimation performance. In a single robot, those indices with bounded covariance can be regarded as connected to the landmark Δ in the graph $\mathscr{O}_i$, according to Proposition 1. Considering the incoming communication information in Proposition 2, the bounded indices are those weakly connected to Δ in the graph $(\Omega^*, \bigcup_{j \in C_i \cup \{i\}} E_{\mathscr{O}_j})$, regardless of edge direction.

In fact, this is not all the indices having bounded covariance, since robot $j \in C_i$ can also receive other incoming information. We define a graph $\mathscr{T}_i$ for each robot $i \in \Omega$ by

$$\mathscr{T}_i = \left(\Omega^*, E_{\mathscr{O}_i} \cup \left(\bigcup_{j \in C_i} E_{\mathscr{T}_j} \right) \right), \tag{20}$$

$$\bigcup_{i \in \Omega} E_{\mathscr{T}_j} = \bigcup_{i \in \Omega} E_{\mathscr{O}_j}. \tag{21}$$

$\mathscr{T}_i$ can be regarded as the collective information obtained by robot i from observation and communication. To be more precise, an edge in $\mathscr{T}_i$ represents an observation relationship, either direct or received from other robots. However, the equation (20) has a trivial solution in which all $\mathscr{T}_i$ is fully-connected. We have to impose one more constraint to avoid such case. The additional equation (21) states that the communication can not invent new observation. Therefore, the total observation pairs in all $\mathscr{T}_i$ should be the same as those in all $\mathscr{O}_i$. For simplicity, we define T_i to be the set of robots weakly connected to Δ in $\mathscr{T}_i$.

Proposition 3. $[\Pi_{s^i,t}]_{\{j\}}$ *is bounded given positive definite initial condition if and only if* $j \in T_i$.

Proof. If $j \in T_i$, we can arrive the conclusion by the direct application of Propositions 1 and 2. For the inverse statement, the proof relies on Lemma 3 in [15]. One can refer to it for details while the complete proof is omitted here.

As Proposition 3 implies, $[\Sigma_{s^i,t}]_{T_i}$ is also bounded.

6 Simulations

In this section, we simulate the proposed algorithm to verify the convergent analysis as well as to compare the performance with other algorithms, including:

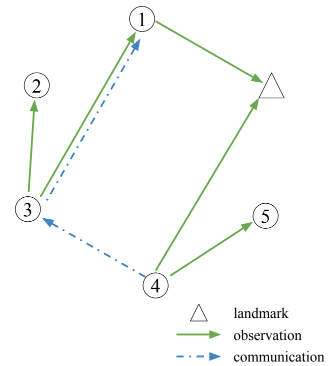

Fig. 1: The observation and communication topologies.

- local state centralized equivalent (LS-Cen) [17, 11]
- local state CI (LS-CI) [3]
- local state block diagonal approximation (LS-BDA) [14]
- global state split CI (GS-SCI) [13]
- proposed global state CI (GS-CI)

In the first part, we use generated data to substantiate the theoretical treatment of the proposed algorithm. In the second part, we apply dataset collected in real-world experimental setup to investigate the localization performance for the whole system.

6.1 Generated Data

Table 1: Number of communication per slot with generated data.

Algorithms	Number of Communication
LS-Cen	23
LS-CI	6
LS-BDA	6
GS-SCI	2
GS-CI	2

We simulate a system with $N = 5$ robots to verify the main propositions of this paper. Robots are moving in a circle with radius 25 m. The velocity input is take uniformly between $[-0.25, 0.25]$ m/s, with $\sigma_{\mathbf{n}_v} = 0.0125$ m/s and $\sigma_{\mathbf{v}} = 0.25$ m/s. The variances of observation are taken as $\sigma_{\mathbf{n}_d} = 0.1$ m and $\sigma_{\mathbf{n}_\phi} = 2$ degree. For LS algorithms, only observation topology is considered as in Fig. 1, and the communication topology is implicitly assumed fully connected, which means that the communica-

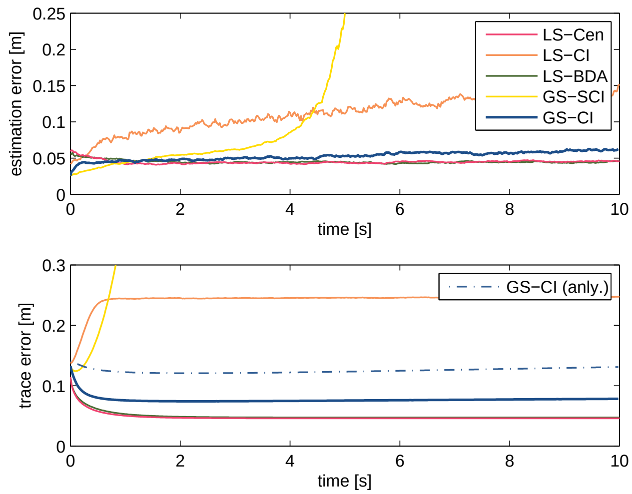

Fig. 2: Simulation result of generated data.

tion after relative observation is assumed perfect. For GS algorithms that separates communication and observation, both topologies follow Fig. 1.

We focus on the estimation in robot 1 and consider two error performance metrics. The estimation error is given by $\sqrt{||\hat{s}^1 - s||_2/N}$, which indicates the error comparing to the ground truth. On the other hand, the trace error is defined as $\sqrt{\mathrm{tr}(\Sigma_{s^1})/N}$, which signifies the convergence of the covariance matrix in EKF.

Based on the simulation result in Fig. 2, LS-CI and GS-SCI are divergent in terms of estimation error, since these two algorithms have no investigation on the boundedness of state estimation covariance matrices. The rest can lead to reasonable estimation results around 0.05 m in estimation error, but the communication cost of each algorithm varies from 23 for LS-Cen to 2 for the proposed GS-CI, as in Table 1. In other words, the proposed GS-CI achieves comparable accuracy with far less communication. The trace error of GS-CI also substantiates the analysis in Section 5: $T_1 = \Omega$, and $\Sigma_{\hat{s}^1}$ is therefore bounded as a result according to Proposition 3.

6.2 Real-World Data

We use UTIAS Multi-Robot Cooperative Localization and Mapping Dataset [12] to compare the performance of algorithms. The UTIAS dataset is a 2D indoor dataset collection consists of 9 individual datasets, and each dataset contains odometry and measurement data from 5 robots, as well as accurate position groundtruth data. We have plotted dataset 6, while the results of all datasets have no significant difference.

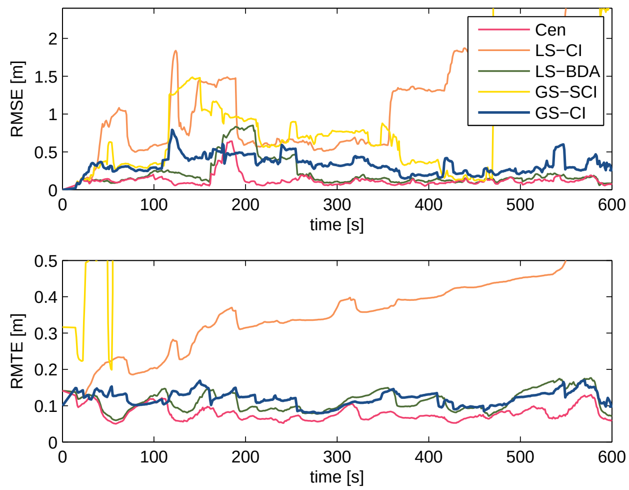

Fig. 3: Simulation result of UTIAS dataset.

In this simulation, we implement a centralized localization as in [17] as a performance benchmark, and focus on the topology constraints on LS and GS algorithms. Specifically, we set that only robots $1,2,3$ can observe landmarks, and the communication only takes place between robots $1,2,3$ and $4,5$ for all algorithms of interest. In other words, robots $4,5$ can only rely on relative observation and communication for localization. To mitigate linearization error, we use orientation data from ground truth. The parameters of propagation and observation errors in the algorithms follow the previous simulation.

Instead of concentrating on the behavior of a single robot as in the previous part, we consider the overall localization performance of all $N = 5$ robots. Two metrics are applied to present the estimation error: the root mean square error (RMSE) is defined as $RMSE = \sqrt{\sum_{i=1}^{N} ||\hat{s}_i^i - s_i||^2/N}$, and $RMTE = \sqrt{\sum_{i=1}^{N} \mathrm{tr}([\Sigma_{s^i}]_i)/N}$ as the definition of root mean trace error (RMTE).

Since only robots 1,2,3 can observe landmarks, only LS-BDA and proposed GS-CI can localize the rest robots in a distributed sense, where the RMSEs of both algorithms are less than 1 m. However, one should notice that the number of communication is half in GS-CI than in LS-BDA, where the communication is bi-directional after relative observation.

7 Conclusions

In this paper, we propose a cooperative localization algorithm for multirobot system with explicit communication mechanism. In the proposed algorithm the covariance intersection is applied in communication update to preserve the consistency of the estimate. We also give the boundedness criteria on covariance matrix of individual robot, which heavily depends on the observation and communication topologies.

The work in this paper is inspiring in several perspectives. In most robotic systems, localization is not the main goal but the underlying requirement for high-level tasks, exploration or self-driving for example. While the proposed algorithm has actions corresponding to the hardware mechanism, one can relate the underlying physical actions to the performance of the targeted tasks relied on localization for further optimization. In addition, while the estimation is one of the fundamental problem in robotics, the estimation scheme used in this paper can be applied to other distributive estimation scenarios beyond localization, such as SLAM problems.

Appendix: Auxiliary Lemmas

Lemma 1. *For $P > 0$ and the index set I, we have* $[P^{-1}]_I > [P]_I{}^{-1}$.

Proof. For convenience, we partition P into blocks where the upper-left submatrix is exactly $[P]_I$. We denote $A = [P]_I$ for simplicity, which gives $P = \begin{bmatrix} A & B \\ B^\top & C \end{bmatrix}$. By matrix inversion lemma, we have

$$P^{-1} = \begin{bmatrix} A^{-1} + A^{-1}B(C - B^\top A^{-1}B)^{-1}B^\top A^{-1} & -A^{-1}B(C - B^\top A^{-1}B)^{-1} \\ -(C - B^\top A^{-1}B)^{-1}B^\top A^{-1} & (C - B^\top A^{-1}B)^{-1} \end{bmatrix}.$$

By assumption, $P^{-1} > 0$, and thus the diagonal submatrix $(C - B^\top A^{-1}B)^{-1} > 0$. In conclusion, we arrive $[P^{-1}]_I = A^{-1} + A^{-1}B(C - B^\top A^{-1}B)^{-1}B^\top A^{-1} > A^{-1} = [P]_I^{-1}$.

Lemma 2. *Suppose that $P > 0$ and $P = \begin{bmatrix} A_{m\times m} & B \\ B^\top & C_{n\times n} \end{bmatrix}$, then $P < P' = \begin{bmatrix} \frac{1}{c}A & 0 \\ 0 & \frac{1}{1-c}C \end{bmatrix}$ for $c \in (0,1)$.*

Proof. Consider a nonzero vector $v^\top = [(1-c)x^\top, cy^\top]$ where $x \in \mathbb{R}^m$ and $y \in \mathbb{R}^n$. Since $P > 0$, $v^\top Pv > 0$, or

$$(1-c)^2 x^\top Ax + c^2 y^\top Cy + c(1-c)y^\top B^\top x + c(1-c)x^\top By > 0.$$

For arbitrary $u^\top = [x^\top, y^\top]$, we then know that

$$Q = \begin{bmatrix} (1-c)^2 A & c(1-c)B \\ c(1-c)B^\top & c^2 C \end{bmatrix} > 0.$$

The result follows by $P' - P = \frac{1}{c(1-c)} JQJ > 0$, where $J = \mathsf{Diag}(I_m, -I_n)$.

References

1. Arambel, P.O., Rago, C., Mehra, R.K.: Covariance intersection algorithm for distributed spacecraft state estimation. In: Proceedings of the 2001 American Control Conference, vol. 6, pp. 4398–4403 vol.6 (2001)
2. Bailey, T., Nieto, J., Guivant, J., Stevens, M., Nebot, E.: Consistency of the EKF-SLAM Algorithm. In: 2006 IEEE/RSJ International Conference on Intelligent Robots and Systems, pp. 3562–3568 (2006)
3. Carrillo-Arce, L.C., Nerurkar, E.D., Gordillo, J.L., Roumeliotis, S.I.: Decentralized multi-robot cooperative localization using covariance intersection. In: 2013 IEEE/RSJ International Conference on Intelligent Robots and Systems, pp. 1412–1417 (2013)
4. Cattivelli, F.S., Sayed, A.H.: Diffusion Strategies for Distributed Kalman Filtering and Smoothing. IEEE Transactions on Automatic Control **55**(9), 2069–2084 (2010)
5. Chen, L., Arambel, P.O., Mehra, R.K.: Estimation under unknown correlation: covariance intersection revisited. IEEE Transactions on Automatic Control **47**(11), 1879–1882 (2002)
6. Hu, J., Xie, L., Zhang, C.: Diffusion Kalman filtering based on covariance intersection. IEEE Transactions on Signal Processing **60**(2), 891–902 (2012)
7. Kailath, T., Sayed, A.H., Hassibi, B.: Linear Estimation. Pearson (2000)
8. Karam, N., Chausse, F., Aufrere, R., Chapuis, R.: Cooperative multi-vehicle localization. In: 2006 IEEE Intelligent Vehicles Symposium, pp. 564–570 (2006)
9. Karam, N., Chausse, F., Aufrere, R., Chapuis, R.: Localization of a group of communicating vehicles by state exchange. In: 2006 IEEE/RSJ International Conference on Intelligent Robots and Systems, pp. 519–524 (2006)
10. Kia, S.S., Rounds, S., Martnez, S.: Cooperative localization under message dropouts via a partially decentralized EKF scheme. In: 2015 IEEE International Conference on Robotics and Automation, pp. 5977–5982 (2015)
11. Kia, S.S., Rounds, S.F., Martnez, S.: A centralized-equivalent decentralized implementation of Extended Kalman Filters for cooperative localization. In: 2014 IEEE/RSJ International Conference on Intelligent Robots and Systems, pp. 3761–3766 (2014)
12. Leung, K.Y., Halpern, Y., Barfoot, T.D., Liu, H.H.: The UTIAS multi-robot cooperative localization and mapping dataset. The International Journal of Robotics Research **30**(8), 969–974 (2011)
13. Li, H., Nashashibi, F., Yang, M.: Split covariance intersection filter: Theory and its application to vehicle localization. IEEE Transactions on Intelligent Transportation Systems **14**(4), 1860–1871 (2013)
14. Luft, L., Schubert, T., Roumeliotis, S.I., Burgard, W.: Recursive decentralized collaborative localization for sparsely communicating robots. In: Proceedings of Robotics: Science and Systems (2016)
15. Mourikis, A.I., Roumeliotis, S.I.: Performance analysis of multirobot cooperative localization. IEEE Transactions on Robotics **22**(4), 666–681 (2006)
16. Reinhardt, M., Noack, B., Arambel, P.O., Hanebeck, U.D.: Minimum Covariance Bounds for the Fusion under Unknown Correlations. IEEE Signal Processing Letters **22**(9), 1210–1214 (2015)
17. Roumeliotis, S.I., Bekey, G.A.: Distributed multirobot localization. IEEE Transactions on Robotics and Automation **18**(5), 781–795 (2002)
18. Saeedi, S., Trentini, M., Seto, M., Li, H.: Multiple-robot simultaneous localization and mapping: A review. Journal of Field Robotics **33**(1), 3–46 (2016)