Linearization Comparison

	Literature	Implementation
Setup	Coordinates: E: an inertial / fixed frame B: a body-fixed frame, of which the positive x direction aligns with the direction of the propeller arm relative to the center of mass of the body. The origin lies at the center of mass of the whole flyer. C: a control frame dependent on the State: ω_{BE}^{B}: the angular velocity of the body relative to the inertial frame expressed in the body frame ω_{PE}^{B}: the angular velocity of the propeller relative to the inertial frame expressed in the body frame I_{B}^{B}: the moment of inertia of the body (without the propeller) with respect to the body's center of mass I_{P}^{B}: the moment of inertia of the propeller with respect to the propeller's center of mass e_{P}^{B}: propeller force direction in the body frame τ_{d}^{B}: air frame drag torque f_P: thrust of the propeller τ_P: torque of the propeller u: the output thrust produced by the attitude controller τ_{mot}: time constant of the motor	
Calculate ω_{PE}^{B}	The propeller's scalar speed Ω with respect to the body is usually controlled by an electronic speed controller, so that $$\boldsymbol{\omega}_{PB}^{B} = (0, 0, \Omega).$$ Note that ω_{PE}^{B} can be decomposed as below: $$\boldsymbol{\omega}_{PE}^{B} = \boldsymbol{\omega}_{PB}^{B} + \boldsymbol{\omega}_{BE}^{B}$$	In order to implement the simulation, an assumption is made: $\omega_{PB}^{B} \gg \omega_{BE}^{B}$. Therefore, $$\omega_{PE}^{B} \approx \omega_{PB}^{B}$$ $\omega_{PB}^{B} = [0; 0; \Omega]$, where Ω is the scalar rotation speed of the propeller,
The relation between the total thrust f and ω_{PE}^{B}	Total thrust f produced by the propeller has the following relation with the angular velocity of the propeller relatively to the earth in the body frame ω_{PE}^{B}. $$f_P = \kappa_f (\boldsymbol{\omega}_{PE}^{B} \cdot \boldsymbol{e}_{P}^{B}) \lvert \boldsymbol{\omega}_{PE}^{B} \cdot \boldsymbol{e}_{P}^{B} \rvert$$	$$f_P = \kappa_f \Omega^2$$
τ_P	$$\tau_P = -\kappa_\tau (\boldsymbol{\omega}_{PE}^{B} \cdot \boldsymbol{e}_{P}^{B}) \lvert \boldsymbol{\omega}_{PE}^{B} \cdot \boldsymbol{e}_{P}^{B} \rvert$$	$$\tau_P = -\kappa_\tau \Omega^2$$
$\dot{f}_P$	$$\dot{f}_P = (f_{position} + u - f_P)/\Delta\tau_{mot}$$	
$\dot{\omega}_{PE}^{B}$	Not specified	$$\dot{f}_P = 2\kappa_f \Omega \dot{\Omega} \quad (6)$$ $$\dot{\Omega} = \frac{f_{position} + u - f_P}{\tau_{mot}} \cdot \frac{1}{2\kappa_f \Omega}$$ $$\dot{\omega}_{PE}^{B} \approx [0; 0; \dot{\Omega}]$$

	$\boldsymbol{\tau}_d^B = -\left\|\boldsymbol{\omega}_{BE}^B\right\| \boldsymbol{K}_d^B \boldsymbol{\omega}_{BE}^B$	
Angular Acceleration of the body in body frame $\dot{\omega}_{BE}^B$	From Euler's Second Law, the angular acceleration of the body in the body frame $\dot{\omega}_{BE}^B$ can be estimated. $$I_B^B \dot{\omega}_{BE}^B + I_P^B \dot{\omega}_{PE}^B + \omega_{BE}^B \times (I_B^B \omega_{BE}^B + I_P^B \omega_{PE}^B) = r_P^B \times e_P^B f_B + e_P^B \tau_P + \tau_d^B$$	
Control Frame C	For convenience a control coordinate system C is introduced which is fixed with respect to the body-fixed coordinate system B and where $$n^B = \pm \frac{\overline{\omega_{BE}^B}}{\left\|\overline{\omega_{BE}^B}\right\|}$$ $$n^C = C^{CB} n^B = [0; 0; 1]$$	
$\dot{n}_{des}^B$		$\dot{n}_{des}^B = -\omega_{BE}^B \times n_{des}^B$
$\dot{n}_{des}^C$	Not specified	$\dot{n}_{des}^C = -\omega_{BE}^C \times n_{des}^C$ or $\dot{n}_{des}^C = -C^{CB} \times \dot{n}_{des}^B$
ω_{BE}^C	$\omega_{BE}^C = C^{CB} \omega_{BE}^B$	
State variable s	Let $n_{des}^C = [\eta_1; \eta_2; \eta_3]$ and $\omega_{BE}^C = [\alpha_1; \alpha_2; \alpha_3]$ $s = (\eta_1, \eta_2, \alpha_1, \alpha_2, \alpha_3, f_P) - (0, 0, 0, 0, \pm \left\|\bar{\boldsymbol{\omega}}_{BE}^B\right\|, \bar{f}_P)$ Notice only x and y components of n_{des}^C are in s since n_{des}^C is a unit vector and the third component can be calculated from the first 2.	
$\dot{s}$	Not Specified	Define a matrix function $$\dot{s} = f(s, u)$$ Then the linearized form is $$\dot{s} = As + Bu$$ where attitude thrust output u has the form$[u]$. The model is linearized around the hover solution, where $$s_{hover} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \quad u_{hover} = [0]$$ Two matrices A and B are obtained by $$A = Jacobian\left(\frac{\partial f}{\partial s}\right)_{hover}$$ $$B = Jacobian\left(\frac{\partial f}{\partial u}\right)_{hover}$$

		$s_{hover} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \quad u_{hover} = [0]$

References:

Zhang, W., Mueller, M. W., & Dandrea, R. (2016). A controllable flying vehicle with a single moving part. *2016 IEEE International Conference on Robotics and Automation (ICRA)*. doi:10.1109/icra.2016.7487499

$$\frac{\partial (f_{position} + u - f_P)/\tau_{mot}}{\partial u} = 1/\tau_{mot}$$