EM SLAM

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February 12, 2019

Notations are very annoying.

1 EM Algorithm

The expectation maximization (EM) algorithm is proposed in [1].

We use k to index iterations of the EM algorithm.

X represents complete data, but Y represents incomplete data.

$$\text{E step:} \qquad Q\left(\eta, \hat{\eta}^{(k)}\right) = \mathsf{E}\left[\log f_X(X;\eta)|Y=y;\hat{\eta}^{(k)}\right], \tag{1}$$

$$\text{M step:} \qquad \hat{\eta}^{(k+1)} = \arg\max_{\eta} Q\left(\eta, \hat{\eta}^{(k)}\right). \tag{2}$$

2 Recursive EM

The recursive EM is developed in [2], and the recursive EM with discount factor is introduced in [3].

In an online algorithm, the time index and the iteration index should match, or at least around the same rate.

The recursive EM can be written as

$$\text{E step:} \qquad L_{k+1}(\eta) = Q_{k+1}\left(\eta, \hat{\eta}^{(k)}\right) = \gamma L_k(\eta) + \mathsf{E}\left[\log f_X(x;\eta)|y_{k+1};\hat{\eta}^{(k)}\right]. \tag{3}$$

$$\text{M step:} \qquad \hat{\eta}^{(k+1)} = \arg\max_{\eta} L_{k+1}(\eta) = \arg\max_{\eta} Q_{k+1}\left(\eta, \hat{\eta}^{(k)}\right). \tag{4}$$

In the equation, γ is the discount factor.

With the Newton second-order approximation, we can combine both steps into the following updates:

$$\hat{\eta}^{(k+1)} = \hat{\eta}^{(k)} + I_{c,k+1}^{-1} S(y_k, \hat{\eta}^{(k)}), \tag{5}$$

$$I_{c,k+1} = \gamma I_{c,k} + \bar{I}_c(\hat{\eta}^{(k)}), \tag{6}$$

where

$$S(y_k, \eta) = \nabla_\eta \log f_Y(y_k; \eta), \tag{7}$$

$$\bar{I}_c(\eta) = -\,\mathsf{E}\left[\nabla_\eta^2 \log f_X(x_k; \eta)\right]. \tag{8}$$

The proof is omitted.

Since the above derivation relies on the work in [4], where $x_{1:k}$ and $y_{1:k}$ are assumed independent. We can first consider a stationary robots observing several times (map construction), and add the mobility later.

3 Application on SLAM Problem

The EM method for the SLAM problem is formulated in [5], but the SLAM algorithm is basically offline.

In the SLAM scenario, the complete data X is the spatial state S_t and the corresponding observation O_t.

Under the setting of EKF, we assume that all the distributions are Gaussian.

The probability density function of complete data at the instance t is given by

$$f_{S_t,O_t}(s_t, o_t; \eta) = f_{S_t|O_t}(o_t|s_t; \eta) f_{S_t}(s_t). \tag{9}$$

The distribution of $f_{S_t}(s_t)$ is derived from $f_{S_{t-1}}(s_{t-1})$, as in the the KF.

The Q function is given by

$$\begin{aligned} Q\left(\eta, \hat{\eta}^{(k)}\right) &= \mathsf{E}\left[\log f_{S_t,O_t}(S_t, o_t; \eta) | O_t = o_t, \hat{\eta}^{(k)}\right] \\ &= c - \frac{1}{2}||o_t - h(S_t; \eta)||_{R^{-1}}^2 \end{aligned}$$

is easily obtained by smoothing.

References

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